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A Deterministic Inventory Model having Exponential Decreasing and Ramp Type Demand with Deterioration under Partial Backlogging

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Abstract

The Present Work is done by taking Demand as Exponentially Decreasing in one case and Ramp Type in other. Deterioration Rate is considered as constant in one case and Weibull Distributed in other. Shortages are allowed with Partial Backlogging in both of the cases.

Keywords:- Inventory Model, Demand, Deterioration, Backlogging, Shortage.

1 Introduction

Inventory management plays a vital role in the smooth and efficient functioning of organizations and industries. The effective management of inventory requires the development of suitable mathematical models that account for various influencing factors. Over the years, researchers from different fields have proposed numerous inventory models under diverse assumptions. Ibe and Ogbeide investigated an inventory system characterized by exponentially increasing demand and a constant rate of deterioration. Shelly and Kumar formulated inventory models incorporating polynomial demand, considering variable deterioration in one study and constant deterioration in the other. In another study, Shelly and Kumar developed an inventory model based on general forms of demand and deterioration. They further investigated inventory systems by incorporating variable holding costs in their models. Wu, Lin, Tan, and Lee proposed an EOQ model in which the demand rate was assumed to follow a ramp-type pattern.

This Model is developed by taking Exponentially Decreasing Demand in the Paper by Ibe and Ogbeide and by taking the case of Partial Backlogging in the Paper by Wu, Lin, Tan and Lee. Methodology is adopted from the previous work and from some of the cited articles.

2 Assumptions and Notations

- Per unit Holding Cost(HC) is assumed as $h(t) = h + \alpha t$ for 1st Case, where $\alpha > 0$ is constant and C_h for 2nd Case.
- Per unit Shortage Cost(SC) is assumed as constant C_s and Per unit Deterioration Cost(DC) is assumed as constant C_d .
- Per unit Lost Sale Cost(LSC) is assumed as constant C_{ls} and Per unit Backlogging Cost(BC) is assumed as C_b .
- Unit Cost of an item is assumed as constant C_1 and Ordering Cost(OC) is assumed as constant A_1 .
- Inventory is taken as $I(t)$ for any time t and this Model is created for single item.
- Demand Rate $D(t)$ is taken as
 - Exponentially Decreasing i.e.

$$D(t) = ae^{-bt}$$
 , where $a, b > 0$ and are constants .
 - Ramp Type Function i.e.

$$D(t) = D_0[t - (t - \mu)H(t - \mu)], D_0 > 0$$

where $H(t - \mu)$ is Heaviside's function given by

$$H(t - \mu) = \begin{cases} 1; & t \geq \mu \\ 0; & t < \mu \end{cases}$$

- Deterioration Rate is assumed as
 - Constant i.e.

$$\theta(t) = \theta_0$$

where θ_0 is constant parameter.

- Weibull Distributed Deterioration i.e.

$$\theta(t) = \alpha\beta t^{\beta-1}$$

where α, β are constant parameters.

- Shortages are allowed with Partially Backlogging.
- Backlogging Rate is assumed as
 - $e^{-\delta t}$,
 - $\frac{1}{1+\Delta(T-t)}$,
 - where Δ is very small & Positive Parameter.
- Inventory level is assumed as Zero at time t_0 i.e. $I(t_0) = 0$.
- P_c is the Purchasing Cost, which is fixed.
- O_c is the Ordering Cost, which is fixed.

3 Analysis of Model

In the Current System the level Inventory is greatest at the beginning i.e. $I(0) = S$, then it starts reducing & then it reaches to 0 at t_0 i.e. $I(t_0) = 0$ and then shortages occurs with Partial Backlogging.

3.1 When Demand is considered as Exponentially Decreasing with Constant Deterioration under Partial Backlogging

For current work, we have taken Exponentially Decreasing in the Paper by Ibe and Ogbeide and the methodology is adopted from the cited work and our previous work. Level of Inventory at any time is $I(t)$ and time interval is $0 \leq t_0 \leq T$. Inventory at beginning is S and at time t_0 level of Inventory becomes Zero. Shortages occur from the period t_0 to T that are Backlogged Partially. System of Equations representing this Inventory Model are as follow:-

$$\frac{dI(t)}{dt} + \theta_0 I(t) = -ae^{-bt}$$

and

$$\frac{dI(t)}{dt} = -ae^{-(b+\delta)t}$$

Solution of diff. eqn. (1) is

$$I(t)e^{\theta_0 t} = -\frac{ae^{(\theta_0-b)t}}{\theta_0-b} + C_1$$

Using $I(t_0) = 0$, Also $I(0) = S \Rightarrow C_1 = S$

$$\begin{aligned} 0 &= -\frac{ae^{(\theta_0-b)t_0}}{\theta_0-b} + C_1 \\ \Rightarrow C_1 &= \frac{ae^{(\theta_0-b)t_0}}{\theta_0-b} = S \end{aligned}$$

Using Value of C_1 in (3),

$$I(t)e^{\theta_0 t} = \frac{ae^{(\theta_0-b)t_0}}{\theta_0-b} - \frac{ae^{(\theta_0-b)t}}{\theta_0-b}$$

Hence

$$I(t) = \frac{a}{\theta_0-b} [e^{(\theta_0-b)t_0-\theta_0 t} - e^{-bt}]$$

Solution of diff. eqn. (2) is

$$\begin{aligned} I(t) &= -a \frac{e^{-(b+\delta)t}}{-(b+\delta)} + C_2 \\ &= a \frac{e^{-(b+\delta)t}}{(b+\delta)} + C_2 \end{aligned}$$

Using $I(t_0) = 0$,

$$\begin{aligned} 0 &= a \frac{e^{-(b+\delta)t_0}}{(b+\delta)} + C_2 \\ \Rightarrow C_2 &= -a \frac{e^{-(b+\delta)t_0}}{(b+\delta)} \end{aligned}$$

Using Value of C_2 in (5),

$$I(t) = \frac{a}{b+\delta} [e^{-(b+\delta)t} - e^{-(b+\delta)t_0}]$$

Inventory Holding Cost is calculated as

$$\begin{aligned} HC &= \int_0^{t_0} h(t)I(t)dt \\ &= \int_0^{t_0} (h + \alpha t)I(t)dt \\ &= \frac{ha}{\theta_0-b} \left(\frac{e^{(\theta_0-b)t_0} - e^{-bt_0}}{\theta_0} + \frac{e^{-bt_0} - 1}{b} \right) \\ &\quad + \frac{\alpha a}{\theta_0-b} \left[\frac{e^{(\theta_0-b)t_0}}{\theta_0^2} - e^{-bt_0} \left(\frac{t_0\theta_0 + 1}{\theta_0^2} \right) + e^{-bt_0} \left(\frac{t_0b + 1}{b^2} \right) - \frac{1}{b^2} \right] \end{aligned}$$

Shortage Cost is calculated as

$$\begin{aligned} SC &= C_s \int_{t_0}^T -I(t)dt \\ &= -\frac{C_s a}{b+\delta} \int_{t_0}^T [e^{-(b+\delta)t} - e^{-(b+\delta)t_0}] dt \\ &= -\frac{C_s a}{b+\delta} \left[\frac{1}{b+\delta} (e^{-(b+\delta)t_0} - e^{-(b+\delta)T}) - e^{-(b+\delta)t_0}(T - t_0) \right] \end{aligned}$$

Lost Sale Cost is calculated as

$$\begin{aligned} LSC &= C_{ls} \int_{t_0}^T (1 - e^{-\delta t}) (ae^{-bt}) dt \\ &= aC_{ls} \int_{t_0}^T e^{-bt} dt - aC_{ls} \int_{t_0}^T e^{-(b+\delta)t} dt \\ &= \frac{aC_{ls}}{b+\delta} [e^{-(b+\delta)T} - e^{-(b+\delta)t_0}] - \frac{aC_{ls}}{b} (e^{-bT} - e^{-bt_0}) \end{aligned}$$

Purchase Cost is calculated as

$$\begin{aligned}
 PC &= C_1 \left(S + \int_{t_0}^T e^{-\delta t} D(t) dt \right) \\
 &= C_1 S - \frac{C_1 a}{b + \delta} [e^{-(b+\delta)T} - e^{-(b+\delta)t_0}]
 \end{aligned}$$

Total Inventory Cost(TC) is Calculated as $TC = OC + PC + HC + SC + LSC$

$$\begin{aligned}
 TC &= A_1 + C_1 S - \frac{C_1 a}{b + \delta} [e^{-(b+\delta)T} - e^{-(b+\delta)t_0}] \\
 &+ \frac{ha}{\theta_0 - b} \left(\frac{e^{(\theta_0 - b)t_0} - e^{-bt_0}}{\theta_0} + \frac{e^{-bt_0} - 1}{b} \right) \\
 &+ \frac{\alpha a}{\theta_0 - b} \left[\frac{e^{(\theta_0 - b)t_0}}{\theta_0^2} - e^{-bt_0} \left(\frac{t_0 \theta_0 + 1}{\theta_0^2} \right) + e^{-bt_0} \left(\frac{t_0 b + 1}{b^2} \right) - \frac{1}{b^2} \right] \\
 &- \frac{C_s a}{b + \delta} \left[\frac{1}{b + \delta} (e^{-(b+\delta)t_0} - e^{-(b+\delta)T}) - e^{-(b+\delta)t_0} (T - t_0) \right] \\
 &+ \frac{a C_{ls}}{b + \delta} [e^{-(b+\delta)T} - e^{-(b+\delta)t_0}] - \frac{a C_{ls}}{b} (e^{-bT} - e^{-bt_0})
 \end{aligned}$$

For minimization of Total Inventory Cost/Time, optimized values t_0 and T can be obtained by solving the equations $\frac{\partial TC}{\partial t_0} = 0$ and $\frac{\partial TC}{\partial T} = 0$ simultaneously, provided the equations $\left(\frac{\partial^2 TC}{\partial t_0^2}\right) \left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_0 \partial T}\right)^2 > 0$ and $\left(\frac{\partial^2 TC}{\partial t_0^2}\right) > 0$ are satisfied by (6).

3.2 When Demand is Ramp Type with Weibull Deterioration under Partial Backlogging

For current work, we have taken the case of Partial Backlogging in the Paper by Wu, Lin, Tan and Lee and the methodology is adopted from the cited work and our previous work. Level of Inventory at any time is $I(t)$ and time interval is $0 \leq t_0 \leq T$. Inventory at beginning is S and at time t_0 level of Inventory becomes Zero. Shortages occur from the period t_0 to T that are Backlogged Partially. System of Equations representing this Inventory Model are as follow:-

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t)$$

and

$$\frac{dI(t)}{dt} = -\frac{D(t)}{1 + \Delta(T - t)}$$

By assumptions 6 and 7 in section 2 and by considering $\mu < t_0$, the eqn^s (1) and (2) becomes

$$\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -D_0 t; 0 \leq t \leq \mu$$

and

$$\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -D_0 \mu; \mu \leq t \leq t_0$$

with

Boundary

Condition

$$I(t_0) = 0.$$

&

$$\frac{dI(t)}{dt} = -\frac{D_0 \mu}{1 + \Delta(T - t)}; t_0 \leq t \leq T$$

with

Initial

Condition

$$I(t_0) = 0.$$

Respective Solutions of diff. eqns. (9), (10) and (11) using the conditions $I(t_0) = 0$ and $I(0) = S$ are

$$I(t) = Se^{-\alpha t^\beta} - D_0 e^{-\alpha t^\beta} \int_0^t x e^{\alpha x^\beta} dx; 0 \leq t \leq \mu$$

$$I(t) = Se^{-\alpha t^\beta} - D_0 e^{-\alpha t^\beta} \left[\int_0^\mu x e^{\alpha x^\beta} dx - \mu \int_\mu^t e^{\alpha x^\beta} dx \right]; \mu \leq t \leq t_0$$

and

$$I(t) = -D_0 \mu (t - t_0) + \Delta D_0 \mu \left[T(t - t_0) - \frac{1}{2} (t^2 - t_0^2) \right]; t_0 \leq t \leq T$$

where

$$S = D_0 \left[\int_0^\mu x e^{\alpha x^\beta} dx + \mu \int_\mu^{t_0} e^{\alpha x^\beta} dx \right]$$

Inventory Holding Cost is calculated as

$$\begin{aligned} HC &= C_h \int_0^{t_0} I(t) dt \\ &= C_h \int_0^\mu \left(Se^{-\alpha t^\beta} - D_0 e^{-\alpha t^\beta} \int_0^t x e^{\alpha x^\beta} dx \right) dt \\ &\quad + C_h \int_\mu^{t_0} \left(D_0 \mu e^{-\alpha t^\beta} \int_t^{t_0} e^{\alpha x^\beta} dx \right) dt \end{aligned}$$

Shortage Cost is calculated as

$$\begin{aligned} SC &= C_s \int_{t_0}^T -I(t) dt \\ &= \frac{1}{2} C_s D_0 \mu (T - t_0)^2 (1 + T\Delta) - \frac{D_0 C_s \mu \Delta}{2} \left(\frac{T^3}{3} - t_0^2 T + \frac{2t_0^3}{3} \right) \end{aligned}$$

Deterioration Cost is calculated as

$$\begin{aligned} DC &= C_d \left[S - \int_0^{t_0} D(t) dt \right] \\ &= C_d \left[S - \int_0^\mu D_0 t dt - \int_\mu^{t_0} D_0 \mu dt \right] \\ &= C_d D_0 \left(\int_0^\mu x e^{\alpha x^\beta} dx + \mu \int_\mu^{t_0} e^{\alpha x^\beta} dx \right) - \frac{1}{2} D_0 C_d \mu^2 - C_d D_0 \mu (t_0 - \mu) \end{aligned}$$

Backlogging Cost is calculated as

$$\begin{aligned} BC &= C_b \int_{t_0}^T \left(1 - \frac{1}{(1 + \Delta(T - t))} \right) D(t) dt \\ &= \frac{C_d D_0 \mu \Delta}{2} (T - t_0)^2 \end{aligned}$$

Total Inventory Cost(TC) is Calculated as $TC = HC + SC + DC + BC$

$$\begin{aligned}
 TC = & C_h \int_0^\mu \left(S e^{-\alpha t^\beta} - D_0 e^{-\alpha t^\beta} \int_0^t x e^{\alpha x^\beta} dx \right) dt \\
 & + C_h \int_\mu^{t_0} \left(D_0 \mu e^{-\alpha t^\beta} \int_t^{t_0} e^{\alpha x^\beta} dx \right) dt \\
 & + \frac{1}{2} C_s D_0 \mu (T - t_0)^2 (1 + T\Delta) - \frac{D_0 C_s \mu \Delta}{2} \left(\frac{T^3}{3} - t_0^2 T + \frac{2t_0^3}{3} \right) \\
 & + C_d D_0 \left(\int_0^\mu x e^{\alpha x^\beta} dx + \mu \int_\mu^{t_0} e^{\alpha x^\beta} dx \right) - \frac{1}{2} D_0 C_d \mu^2 - C_d D_0 \mu (t_0 - \mu) \\
 & + \frac{C_d D_0 \mu \Delta}{2} (T - t_0)^2
 \end{aligned}$$

For minimization of Total Inventory Cost/Time, optimized values t_0 and T can be obtained by solving the equations $\frac{\partial TC}{\partial t_0} = 0$ and $\frac{\partial TC}{\partial T} = 0$ simultaneously, provided the equations $\left(\frac{\partial^2 TC}{\partial t_0^2}\right) \left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_0 \partial T}\right)^2 > 0$ and $\left(\frac{\partial^2 TC}{\partial t_0^2}\right) > 0$ are satisfied by (16).

4 Numerical Illustration and Sensitivity Analysis

The numerical illustrations below follow the structure of the model and are carried out using the corresponding total-cost expressions and constrained numerical optimization. Percentage changes in the parameters are taken as -50% , -25% , 0% , $+25\%$ and $+50\%$, with all other parameters kept at their base values. The reported changes in the decision variables and average total cost are measured with respect to the base solution.

4.1 Case I: Exponentially Decreasing Demand with Constant Deterioration

For Case I, the following parameter values are considered:

$$\begin{aligned}
 h = 50, \quad \alpha = 5, \quad C_s = 80, \quad C_{ls} = 30, \quad C_1 = 10, \quad A_1 = 1000, \\
 a = 100, \quad b = 0.02, \quad \theta_0 = 0.50, \quad \delta = 0.30.
 \end{aligned}$$

Here, α denotes the coefficient of the time-dependent holding cost $h(t) = h + \alpha t$. The average total cost is taken as $ATC = TC/T$, consistent with the minimization statement preceding the optimality conditions. Numerical solution of the two first-order conditions gives

$$t_0^* = 0.7121, \quad T^* = 1.4029,$$

with

$$S = 293.2317, \quad TC = 7839.7318, \quad ATC = 5588.1411.$$

The corresponding cost components are approximately $HC = 1451.6071$, $SC = 1413.7327$, $LSC = 548.6048$, $PC = 3425.7872$ and $OC = 1000$. The Hessian matrix of ATC with respect to (t_0, T) at the reported solution has positive eigenvalues (approximately 1670.25 and 13178.59), supporting a local minimum.

Table 1: Sensitivity analysis for Case I.

Parameter	Change	New Value	% Δt_0^*	% ΔT^*	% ΔATC
h	-50%	25.00	+38.63	+13.58	-11.98
	-25%	37.50	+15.79	+5.50	-5.14
	+25%	62.50	-11.71	-4.04	+4.05
	+50%	75.00	-20.80	-7.13	+7.34
C_s	-50%	40.00	-13.28	+24.40	-13.03
	-25%	60.00	-5.19	+8.91	-5.32
	+25%	100.00	+3.65	-5.86	+3.92

	+50%	120.00	+6.39	-10.03	+6.95
C_{ls}	-50%	15.00	+1.54	+5.14	-3.62
	-25%	22.50	+0.70	+2.47	-1.78
	+25%	37.50	-0.58	-2.29	+1.72
	+50%	45.00	-1.04	-4.42	+3.39
C_1	-50%	5.00	-15.14	-20.70	-23.51
	-25%	7.50	-7.03	-10.75	-11.30
	+25%	12.50	+6.25	+9.63	+10.60
	+50%	15.00	+11.93	+19.00	+20.62
a	-50%	50.00	+13.57	+17.17	-44.12
	-25%	75.00	+4.77	+5.90	-21.90
	+25%	125.00	-3.00	-3.65	+21.75
	+50%	150.00	-5.06	-6.13	+43.42
θ_0	-50%	0.25	+43.72	+44.20	+21.06
	-25%	0.375	+15.76	+15.16	+7.50
	+25%	0.625	-10.51	-9.56	-4.76
	+50%	0.75	-18.20	-16.27	-8.03

4.2 Discussion of Results for Case I

The sensitivity results in Table 1 indicate the following:

1. An increase in holding cost h decreases both optimal times t_0^* and T^* , while the average total cost increases.
2. An increase in shortage cost C_s increases t_0^* but decreases T^* . The average total cost increases because shortages become more expensive.
3. An increase in lost-sale cost C_{ls} produces a small decrease in both optimal times and an increase in average total cost.
4. The unit purchasing cost C_1 has a comparatively strong positive effect on both t_0^* and T^* and on average total cost.
5. The demand-scale parameter a has a pronounced effect on average total cost: increasing a increases the cost and reduces both optimal times.
6. Increasing the constant deterioration parameter θ_0 decreases the optimal times in the numerical range considered, while the average total cost decreases. This direction is a numerical property of the present cost structure and parameter set.

4.3 Graphical Representation for Case I

The following figures display the numerical variations of the selected parameters with t_0^* , T^* and ATC .

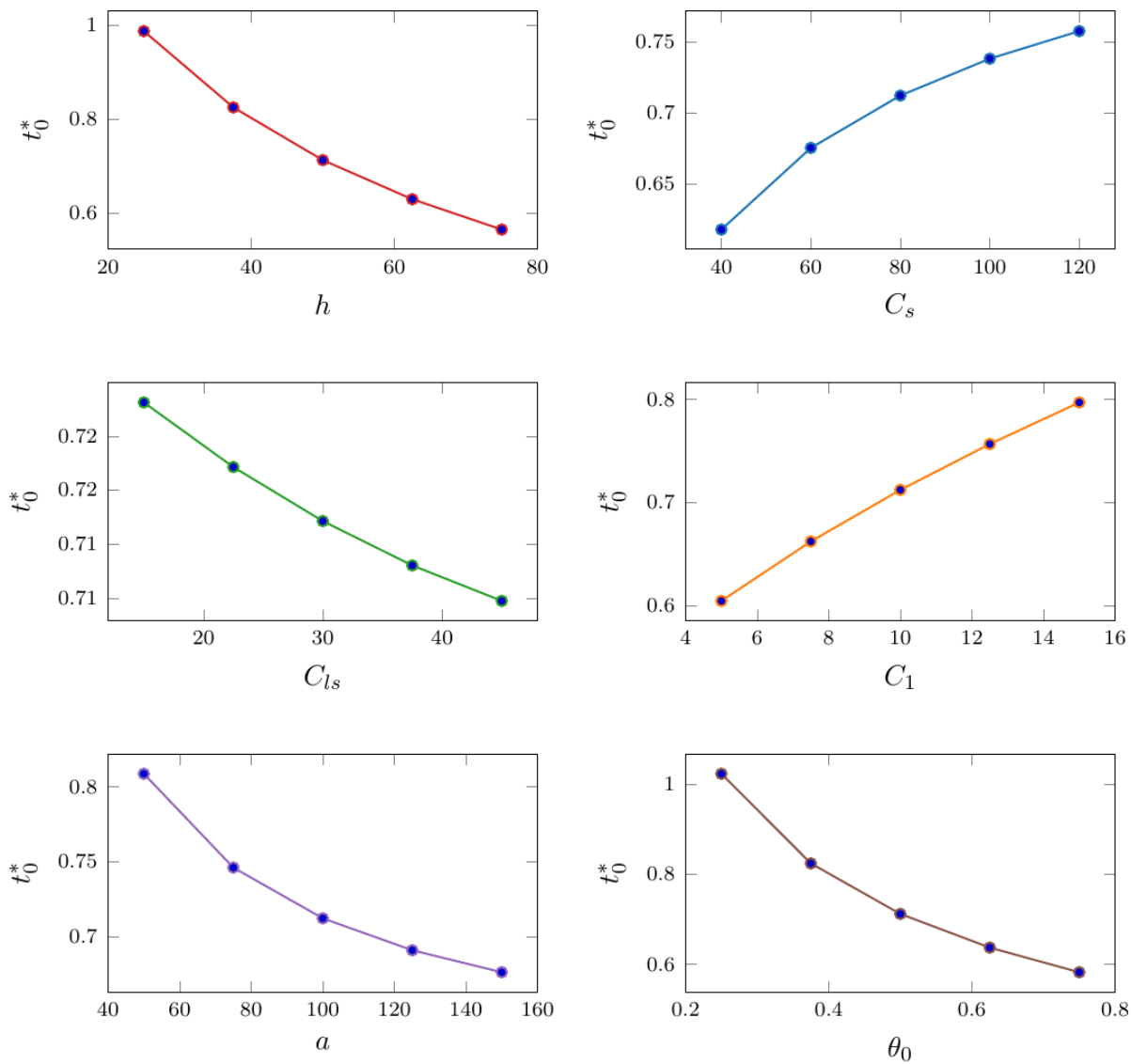


Figure 1: Variation of selected Case I parameters with t_0^* .

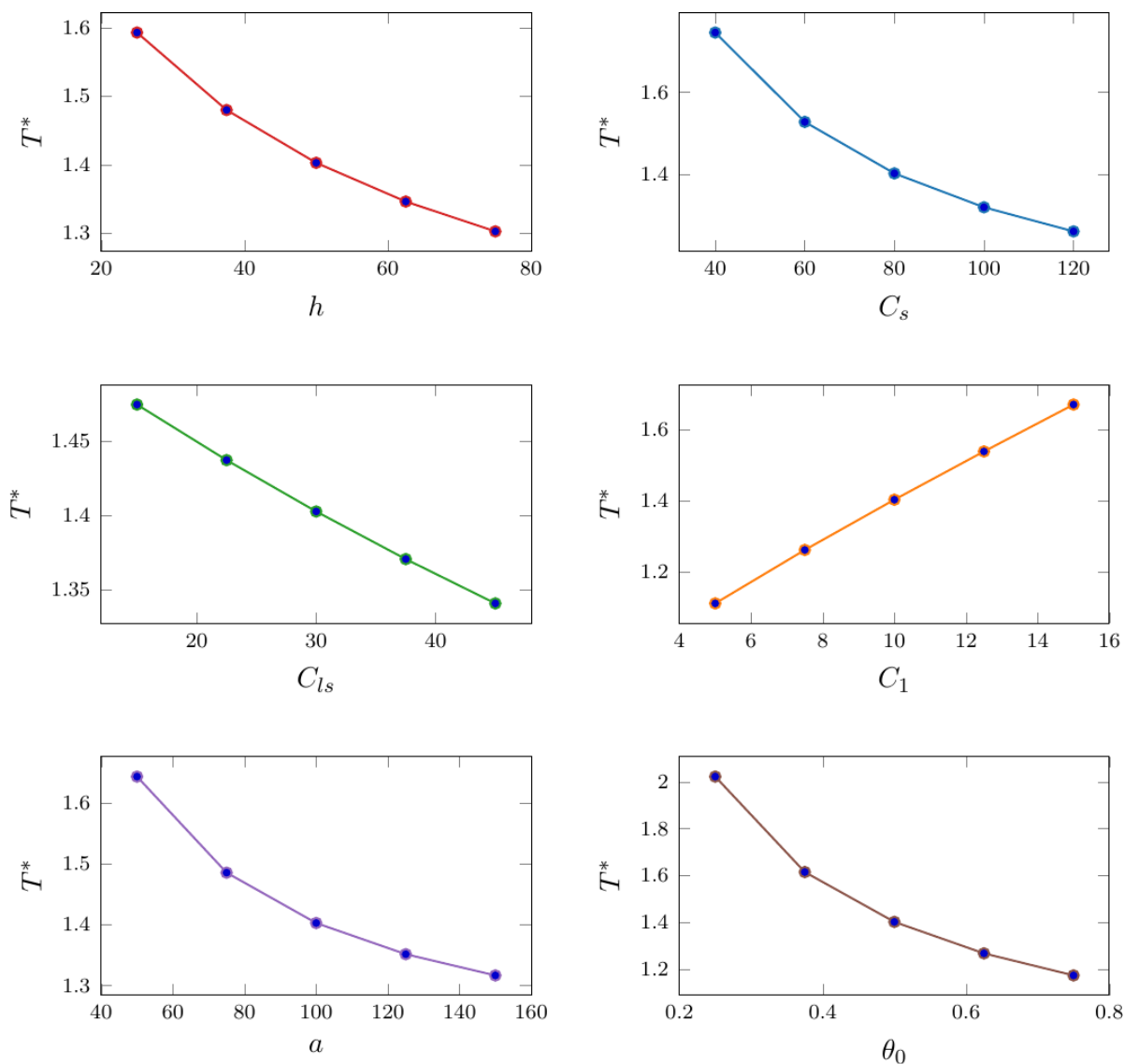


Figure 2: Variation of selected Case I parameters with T^* .

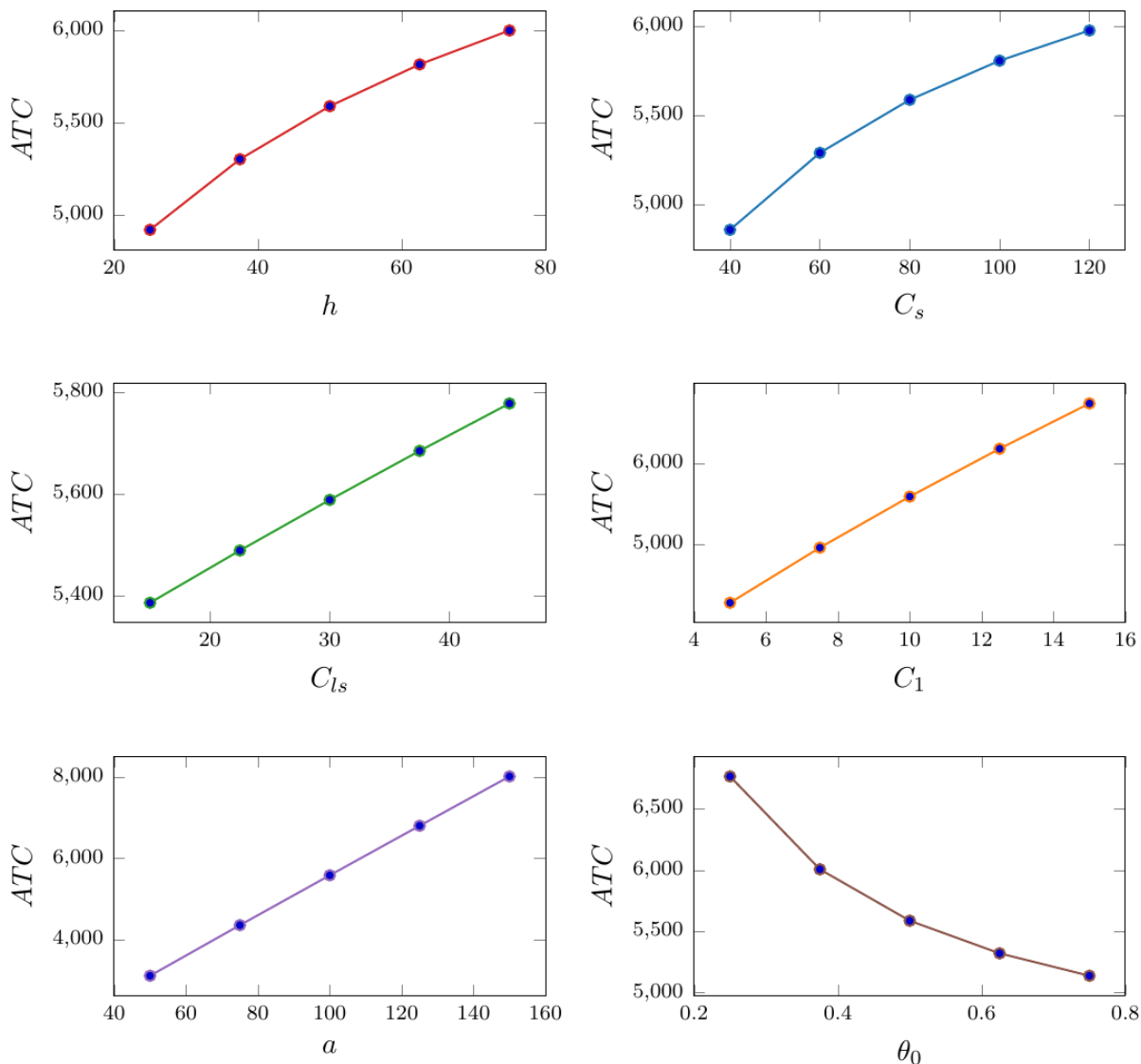


Figure 3: Variation of selected Case I parameters with average total cost.

4.4 Case II: Ramp Type Demand with Weibull Deterioration

For Case II, the demand breakpoint is fixed at $\mu = 0.50$ and the following values are used:

$$C_h = 10, \quad C_s = 100, \quad C_d = 10, \quad C_b = 5, \quad D_0 = 100, \\ \alpha = 0.50, \quad \beta = 1.50, \quad \Delta = 0.10, \quad A_1 = 1000, \quad C_1 = 10.$$

The assumptions list fixed purchasing and ordering costs, although these terms are not explicitly retained in the displayed Case II expression for TC . For numerical consistency with those stated assumptions, the illustration uses

$$TC_2^* = HC + SC + DC + BC + A_1 + C_1S.$$

Also, the backlogging-cost calculation uses C_b , as defined in Assumption 3, rather than C_d . Minimizing $ATC_2 = TC_2^*/T$ subject to $0 < \mu < t_0 < T$ gives

$$t_0^* = 0.8971, \quad T^* = 1.2084,$$

with $S = 40.5784$, $TC_2^* = 1948.9254$ and

$$ATC_2 = 1612.7916.$$

The approximate cost components are $HC = 212.3852$, $SC = 247.3137$, $DC = 82.2308$, $BC = 1.2114$, $PC = 405.7843$ and $OC = 1000$. The two eigenvalues of the numerical Hessian of ATC_2 with respect to (t_0, T) are approximately 750.95 and 9736.72, supporting a local minimum.

Table 2: Sensitivity analysis for Case II.

Parameter	Change	New Value	% Δt_0^*	% ΔT^*	% ΔATC
C_h	-50%	5.00	+13.03	+8.15	-6.09
	-25%	7.50	+6.07	+3.78	-2.88
	+25%	12.50	-5.34	-3.32	+2.59
	+50%	15.00	-10.10	-6.28	+4.93
C_s	-50%	50.00	-9.91	+11.77	-10.51
	-25%	75.00	-3.62	+4.23	-3.95
	+25%	125.00	+2.35	-2.72	+2.65
	+50%	150.00	+4.01	-4.64	+4.56
C_d	-50%	5.00	+6.01	+3.90	-2.25
	-25%	7.50	+2.87	+1.85	-1.09
	+25%	12.50	-2.62	-1.69	+1.02
	+50%	15.00	-5.05	-3.25	+1.99
D_0	-50%	50.00	+32.78	+35.09	-28.30
	-25%	75.00	+13.40	+13.92	-13.02
	+25%	125.00	-10.22	-10.33	+11.49
	+50%	150.00	-18.51	-18.47	+21.80
α	-50%	0.25	+23.58	+15.82	-6.86
	-25%	0.375	+9.94	+6.60	-3.13
	+25%	0.625	-7.71	-5.04	+2.68
	+50%	0.75	-13.91	-9.06	+5.03
β	-50%	0.75	+3.51	+3.62	+4.05
	-25%	1.125	+0.88	+1.10	+1.79
	+25%	1.875	-0.17	-0.48	-1.43
	+50%	2.25	-0.02	-0.66	-2.59

4.5 Discussion of Results for Case II

The sensitivity results in Table 2 show that:

1. Increasing the holding cost C_h reduces both optimal times and increases average total cost.
2. Increasing the shortage cost C_s increases t_0^* but reduces T^* in the numerical range considered, while average total cost increases.
3. The deterioration cost C_d has a moderate effect on both optimal times and a relatively small effect on average total cost compared with D_0 .
4. The demand-scale parameter D_0 has the largest numerical effect among the selected operating parameters. Its increase decreases both optimal times and increases average total cost.
5. Increasing the Weibull scale parameter α decreases the optimal times and increases average total cost over the chosen range.
6. Changes in the Weibull shape parameter β produce comparatively smaller changes in the optimal decision variables and average total cost than changes in D_0 .

4.6 Graphical Representation for Case II

Figures 4–6 show the corresponding variations for Case II.[fig:case2t0][fig:case2cost]

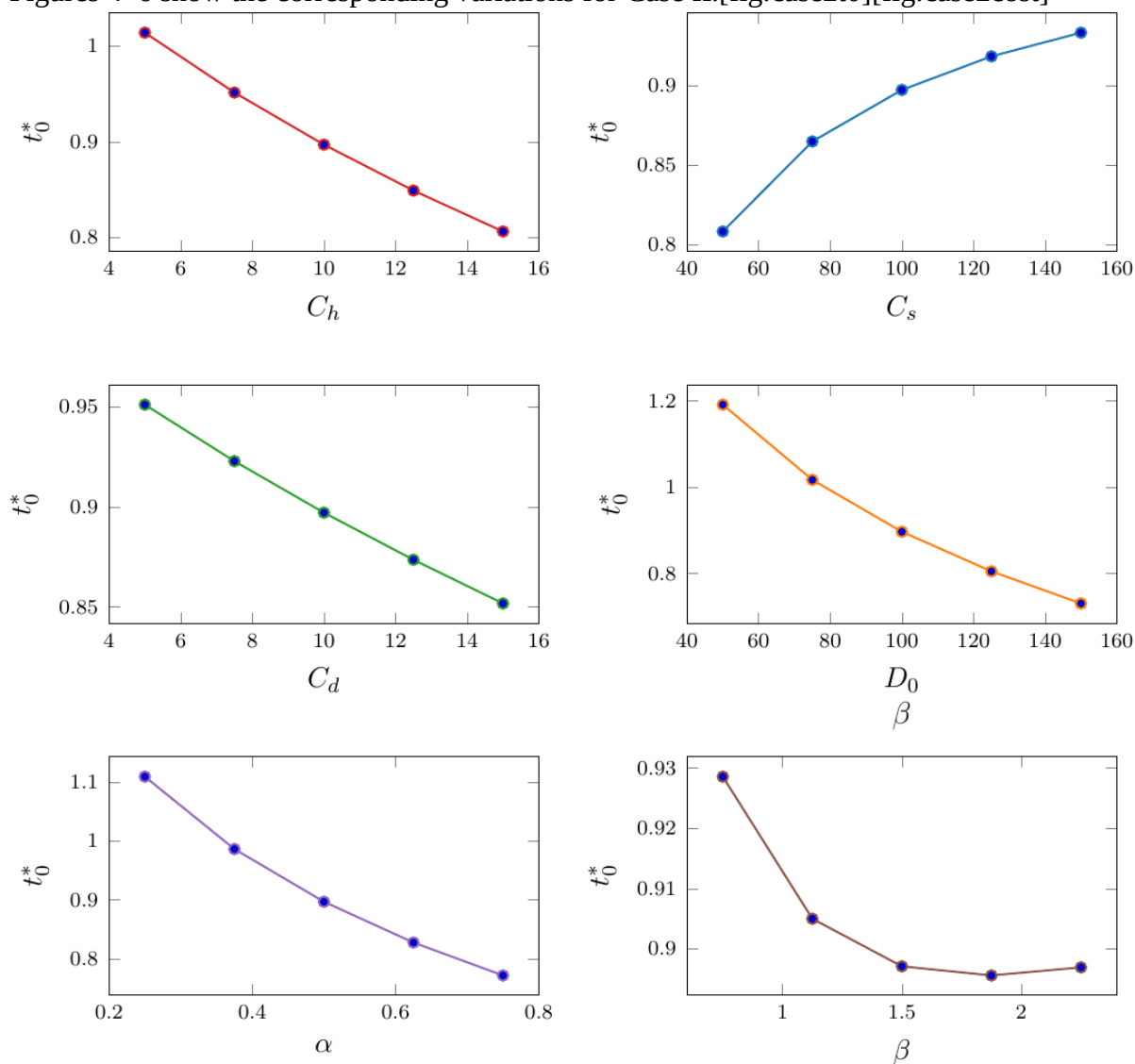


Figure 4: Variation of selected Case II parameters with t_0^* .

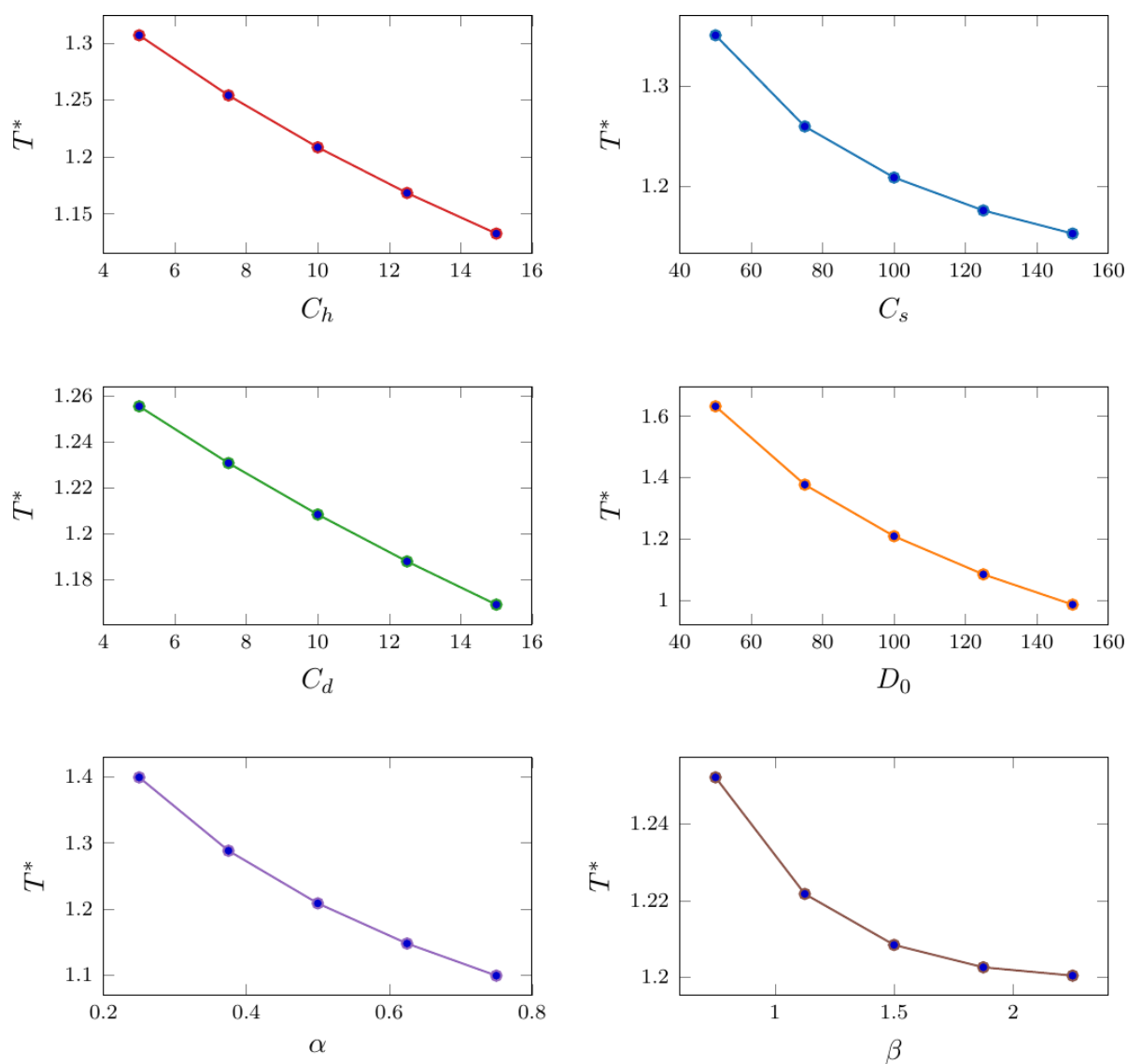


Figure 5: Variation of selected Case II parameters with T^* .

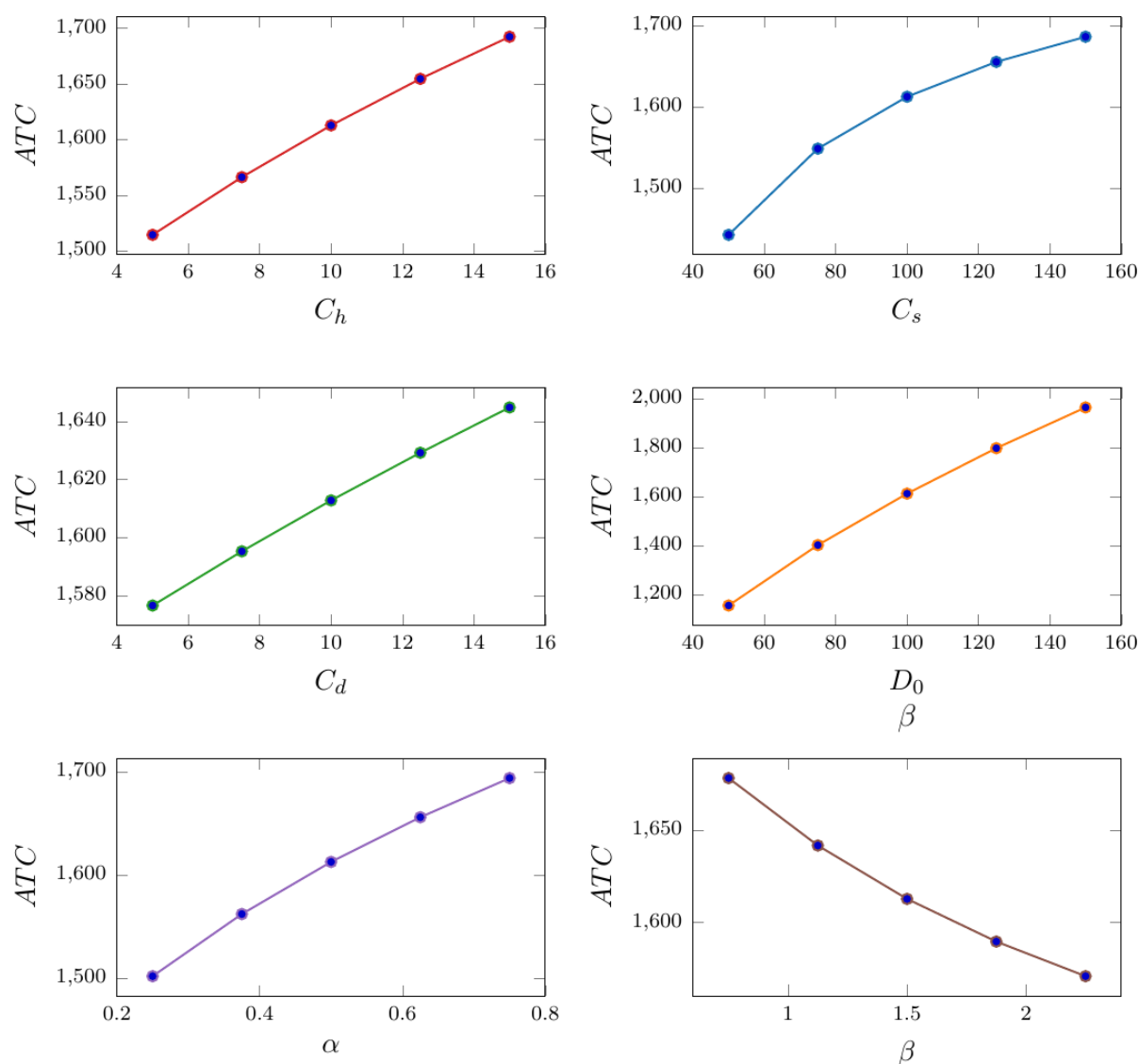


Figure 6: Variation of selected Case II parameters with average total cost.

4.7 Graphical Interpretation

For Case I, the graphs corresponding to h and C_s show a decrease in the optimal cycle-related times as holding or shortage costs increase, although the two decision variables do not necessarily move in the same direction for every parameter. The C_1 and a graphs show comparatively larger changes in the optimal times and average total cost. The C_{ls} graph is relatively flat, indicating a smaller numerical response over the selected range. The θ_0 graph shows a decreasing trend in both optimal times over the selected parameter range.

For Case II, the graphs show that D_0 produces the strongest variation in both optimal times and average total cost among the selected parameters. The graphs for C_h , C_s and C_d display smoother changes, while the β plots are comparatively flatter. The α plots indicate a decrease in both optimal times as the Weibull scale parameter increases, together with an increase in average total cost over the numerical range considered.

4.8 Overall Numerical Interpretation

The two cases demonstrate that the economic consequences of the model parameters depend on the demand and deterioration mechanisms used. In Case I, the unit purchasing cost and demand-scale parameter generate comparatively large changes in average total cost. In Case II, the demand scale

D_0 is particularly influential, whereas the Weibull shape parameter has a smaller numerical effect for the selected values. These observations are parameter-specific numerical findings and should not be interpreted as universal monotonic properties outside the investigated ranges.

5 Conclusion

The Current Work represents Deterministic Inventory Model with Demand as Exponential in one case and Ramp Type in 2nd case. Deterioration Rate as constant in 1st case and as Weibull distributed in other. Shortages are considered as allowed with Partially Backlogging for both the cases. Illustration of the Model is also performed with the use Mathematica and/or MaTLaB by taking a relevant example.

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