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An Inventory Model with Accelerating Exponential Price-Dependent Demand, Weibull Deterioration, Time-Dependent Holding Cost and Partial Backlogging

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Abstract

This paper develops a deterministic single-item inventory model for deteriorating products with accelerating time-dependent demand and price effects. The selling price is prescribed and is calibrated over a six-month interval to represent a moderate increase in product price. Deterioration follows a delayed Weibull law, holding cost is time dependent, and shortages are allowed with waiting-time-dependent partial backlogging. The positive-inventory and shortage differential equations are solved in integral form, and the average total relevant cost is minimized numerically with respect to the positive-inventory duration and replenishment cycle. A numerical example, sensitivity analysis over parameter changes, and graphical interpretation are presented. $\pm 50\%$

Keywords: inventory model; accelerating exponential demand; price-dependent demand; Weibull deterioration; partial backlogging; time-dependent holding cost; sensitivity analysis.

Introduction

Inventory control for deteriorating products is complicated by the simultaneous effects of demand variation, deterioration, shortages, customer waiting behavior and holding costs. Weibull deterioration has been used extensively because its shape parameter permits flexible deterioration patterns. Covert and Philip introduced a Weibull deterioration formulation for EOQ inventory control, and Philip subsequently generalized the model to a three-parameter Weibull setting .

Demand variation has also received substantial attention. Exponential time-varying demand has been considered in deteriorating inventory systems, while partial backlogging distinguishes customers who are willing to wait from those who leave when stock is unavailable. Ouyang, Wu and Cheng considered exponential demand with partial backlogging, while related studies have combined Weibull deterioration with time-dependent demand and partial backlogging .

Price-dependent demand is relevant when the market response changes with the selling price. Previous inventory studies have incorporated price-dependent demand together with deterioration, partial backlogging and joint pricing decisions . In the present formulation, the prescribed selling price contributes positively to the demand component, while the time-dependent market component grows at an accelerating rate. The interaction becomes more pronounced when the time-growth term is accelerating rather than simply exponential in t . Motivated by this observation, the present model employs the nonlinear factor e^{t^2} .

The authors' earlier research has considered polynomial and general demand, constant and variable deterioration, time-dependent holding cost, price-dependent demand, Weibull deterioration and partial backlogging in different settings . The present formulation builds on those themes by

combining an accelerating demand component with a prescribed quadratic price trajectory, delayed Weibull deterioration, time-dependent holding cost and waiting-time-dependent partial backlogging. e^{t^2}

A further feature is the explicit treatment of a six-month price-calibration interval. The calibration is applied to a specified calendar-time interval, after which the resulting price function is used throughout the inventory analysis.

Assumptions and Notations

The model is developed under the following assumptions:

8. A single deteriorating item is considered over an infinite replenishment horizon.
9. Replenishment is instantaneous and the lead time is zero.
10. Each cycle has a positive-inventory period $[0, t_1]$ followed by a shortage period $[t_1, T]$, where $0 < t_1 < T$.
11. Demand is increasing exponentially through an accelerating factor e^{t^2} and is directly related to the increasing selling price through a constant elasticity parameter.
12. The selling price is prescribed rather than optimized and follows a quadratic calendar-time path.
13. Deterioration is negligible during an initial period of length γ and subsequently follows a Weibull rate.
14. Shortages are permitted. A fraction of shortage demand is backlogged and the remainder is lost.
15. The fraction of customers willing to wait is dependent on the waiting time for replenishment.
16. Holding cost per unit per unit time increases linearly with time.
17. Ordering, deterioration, shortage and lost-sales costs are incurred as specified below.

The principal notation is: t = model time, τ = calendar time, t_1 = time at which inventory becomes zero, T = replenishment cycle length, $I_1(t)$ = positive inventory, $I_2(t)$ = shortage inventory, W = maximum on-hand inventory, B = maximum backlog, Q = replenishment quantity, a = demand scale, η = price elasticity, C = price constant, α, β, γ = Weibull parameters, ε = backlogging parameter, h_0, h_1 = holding-cost parameters, K = ordering cost, and c_d, c_s, c_l = deterioration, shortage and lost-sales cost coefficients.

The mathematical assumptions for demand, price, deterioration, backlogging and holding cost are stated explicitly as follows:

$$p(\tau) = \tau^2 - 5\tau + C. \quad (1)$$

With $\tau = 3 + t$,

$$p(t) = t^2 + t + C - 6. \quad (2)$$

The price-dependent accelerating demand is

$$D(t) = ae^{t^2} [p(t)]^\eta = ae^{t^2} (t^2 + t + C - 6)^\eta, \quad \eta > 0. \quad (3)$$

The Weibull deterioration rate is

$$\theta(t) = \begin{cases} 0, & 0 \leq t \leq \gamma, \\ \alpha\beta(t - \gamma)^{\beta-1}, & t > \gamma. \end{cases} \quad (4)$$

The partial-backlogging fraction is

$$b(t, T) = \frac{1}{1 + \varepsilon(T - t)}, \quad \varepsilon > 0. \quad (5)$$

The time-dependent holding-cost rate is

$$h(t) = h_0 + h_1 t. \quad (6)$$

Analysis of Model

Demand and Price Function

Let the relationship between model time and calendar time be

$$\tau = 3 + t. \quad (7)$$

The prescribed selling-price function in calendar time is

$$p(\tau) = \tau^2 - 5\tau + C. \quad (8)$$

Substitution of $\tau = 3 + t$ gives the price in model time as

$$p(t) = (3 + t)^2 - 5(3 + t) + C = t^2 + t + C - 6. \quad (9)$$

The demand rate is assumed to have an accelerating exponential time component and to be directly related to the selling price:

$$D(t, p) = ae^{t^2} p(t)^\eta, \quad a > 0, \quad \eta > 0. \quad (10)$$

Using the price function, the demand rate becomes

$$D(t) = ae^{t^2} (t^2 + t + C - 6)^\eta. \quad (11)$$

For a positive price, the model requires $t^2 + t + C - 6 > 0$ over the relevant cycle.

For the numerical study, the constant is taken as $C = 15$. This choice keeps the selling price positive and increasing over the selected calendar-time interval. In particular, the price changes from 9 to 9.75 between calendar times 3 and 3.5, corresponding to an 8.33% increase. The six-month interval is used only to calibrate a reasonable price level; the optimization itself is carried out over the inventory cycle and does not impose a fixed percentage increase at every cycle.

Weibull Deterioration

The delayed Weibull deterioration rate is defined by

$$\theta(t) = \begin{cases} 0, & 0 \leq t \leq \gamma, \\ \alpha\beta(t - \gamma)^{\beta-1}, & t > \gamma, \end{cases} \quad \alpha > 0, \quad \beta > 0. \quad (12)$$

The corresponding cumulative deterioration exponent is conveniently written as

$$G(t) = \int_0^t \theta(s) ds = \alpha[\max(t - \gamma, 0)]^\beta. \quad (13)$$

For $\beta > 1$, the deterioration rate increases after the delayed onset; for $\beta = 1$ it is constant after onset; and for $0 < \beta < 1$ it decreases after the onset.

Positive Inventory Period and Its Solution

During $0 \leq t \leq t_1$, inventory is positive. Demand and deterioration jointly reduce the stock level, so

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -D(t), \quad 0 \leq t \leq t_1, \quad (14)$$

with boundary condition

$$I_1(t_1) = 0. \quad (15)$$

Using equation (12), the integrating factor is

$$\mu(t) = e^{G(t)}. \quad (16)$$

Multiplying equation (14) by $e^{G(t)}$ gives

$$e^{G(t)} \frac{dI_1(t)}{dt} + e^{G(t)} \theta(t)I_1(t) = -e^{G(t)} D(t). \quad (17)$$

Since $G'(t) = \theta(t)$,

$$\frac{d}{dt} [e^{G(t)} I_1(t)] = -e^{G(t)} D(t). \quad (18)$$

Integrating equation (17) from t to t_1 and applying $I_1(t_1) = 0$ yields

$$I_1(t) = e^{-G(t)} \int_t^{t_1} e^{G(u)} D(u) du. \quad (19)$$

After substituting the explicit demand and $G(u)$,

$$I_1(t) = e^{-G(t)} \int_t^{t_1} e^{G(u)} a e^{u^2} (u^2 + u + C - 6)^n du. \quad (20)$$

The maximum on-hand inventory is therefore

$$W = I_1(0) = e^{-G(0)} \int_0^{t_1} e^{G(u)} D(u) du. \quad (21)$$

Because $\gamma > 0$ in the numerical example, $G(0) = 0$.

Shortage Period and Partial Backlogging

When inventory reaches zero at t_1 , shortages begin. The fraction of demand that is backlogged is assumed to be

$$b(t, T) = \frac{1}{1 + \varepsilon(T-t)}, \quad \varepsilon > 0, \quad t_1 \leq t \leq T. \quad (22)$$

The backlogged fraction therefore varies with the waiting time for replenishment. Indeed,

$$\frac{\partial b(t, T)}{\partial t} = \frac{\varepsilon}{[1 + \varepsilon(T-t)]^2} > 0. \quad (23)$$

The fraction of demand that is lost is

$$1 - b(t, T) = \frac{\varepsilon(T-t)}{1 + \varepsilon(T-t)}. \quad (24)$$

Let $I_2(t)$ denote negative inventory during the shortage period. The shortage equation is

$$\frac{dI_2(t)}{dt} = -b(t, T)D(t) = -\frac{D(t)}{1 + \varepsilon(T-t)}, \quad t_1 \leq t \leq T, \quad (25)$$

with

$$I_2(t_1) = 0. \quad (26)$$

Integrating equation (25) from t_1 to t gives

$$I_2(t) = -\int_{t_1}^t \frac{D(u)}{1 + \varepsilon(T-u)} du. \quad (27)$$

Thus the maximum backlog at the end of the cycle is

$$B = -I_2(T) = \int_{t_1}^T \frac{D(u)}{1 + \varepsilon(T-u)} du. \quad (28)$$

Order Quantity

At the instant of replenishment, the incoming order must replace both the maximum on-hand stock consumed during the positive phase and the outstanding backlog. Consequently,

$$Q = W + B. \quad (29)$$

Using equations (20) and (28),

$$Q = e^{-G(0)} \int_0^{t_1} e^{G(u)} D(u) du + \int_{t_1}^T \frac{D(u)}{1 + \varepsilon(T-u)} du. \quad (30)$$

Cost Components

The holding-cost rate per unit of inventory per unit time is retained in the form used in the earlier model:

$$h(t) = h_0 + h_1 t. \quad (31)$$

Therefore, holding cost per cycle is

$$HC = \int_0^{t_1} (h_0 + h_1 t) I_1(t) dt. \quad (32)$$

The number of units deteriorating during the positive-inventory phase is represented by $\theta(t)I_1(t)$.

Thus deterioration cost is

$$DC = c_d \int_0^{t_1} \theta(t) I_1(t) dt. \quad (33)$$

Shortage cost is charged on the accumulated negative inventory:

$$SC = c_s \int_{t_1}^T [-I_2(t)] dt. \quad (34)$$

Lost-sales demand during the shortage period is $(1 - b(t, T))D(t)$. Hence lost-sales cost is

$$LC = c_l \int_{t_1}^T D(t) \left[1 - \frac{1}{1 + \varepsilon(T-t)} \right] dt. \quad (35)$$

If K denotes the ordering cost per replenishment, the total relevant cost incurred in one cycle is

$$C_{\text{cycle}} = K + HC + DC + SC + LC. \quad (36)$$

Average Total Cost and Optimality Conditions

Since the cycle length is T , the average total cost per unit time is

$$TC(t_1, T) = \frac{K + HC + DC + SC + LC}{T}. \quad (37)$$

The optimal replenishment policy is obtained from

$$\min_{0 < t_1 < T} TC(t_1, T). \quad (38)$$

For an interior solution, the first-order conditions are

$$\frac{\partial TC(t_1, T)}{\partial t_1} = 0, \quad \frac{\partial TC(t_1, T)}{\partial T} = 0. \quad (39)$$

The constraints $0 < t_1 < T$ must also be satisfied. Because the demand function contains e^{t^2} multiplied by a nonlinear price-elasticity term, the resulting integrals generally do not have elementary closed forms. Numerical quadrature is therefore used to evaluate the cost integrals, followed by constrained nonlinear optimization. A positive-definite numerical Hessian at the obtained stationary point is used as a local second-order diagnostic.

Numerical Example and Sensitivity Analysis

The following parameter values are adopted for numerical illustration:

$$\begin{aligned} a &= 10, & \eta &= 0.5, & C &= 15, \\ \alpha &= 0.06, & \beta &= 1.8, & \gamma &= 0.15, \\ \varepsilon &= 0.001, & h_0 &= 0.8, & h_1 &= 0.15, \\ c_d &= 1.2, & c_s &= 1.5, & c_l &= 2, \\ K &= 10. \end{aligned} \quad (40)$$

For these values, the selling-price function remains positive over the relevant cycle. Solving the two first-order optimality equations numerically under $0 < t_1 < T$ gives

$$t_1^* = 0.498554, \quad T^* = 0.785678. \quad (41)$$

Using these optimal values in the inventory and cost expressions gives the numerical results in Table 1.

Numerical values of inventory measures and individual relevant costs.

Measure	Value
Optimal shortage point t_1^*	0.498554
Optimal cycle length T^*	0.785678
Maximum inventory W	16.646351
Maximum backlog B	13.940716
Order quantity Q	30.587067
Ordering cost OC	10.000000
Holding cost HC	3.595287
Deterioration cost DC	0.049560
Shortage cost SC	2.800499
Lost-sales cost LC	0.003734
Total relevant cost per cycle	16.449080

Measure	Value
Average total cost TC^*	20.936153
Hessian eigenvalue 1	42.769987
Hessian eigenvalue 2	196.083306

The total relevant cost per cycle is obtained by adding the five cost components. Thus,

$$C_{\text{cycle}} = 10.000000 + 3.595287 + 0.049560 + 2.800499 + 0.003734 = 16.449080. \quad (42)$$

The corresponding average total cost is

$$TC^* = \frac{16.449080}{0.785678} = 20.936153. \quad (43)$$

Both eigenvalues of the numerical Hessian are positive at the stationary point, giving a local second-order indication of a minimum.

Sensitivity Analysis

The sensitivity analysis is performed by changing one model parameter at a time by -50% , -25% , $+25\%$ and $+50\%$, while all other parameters are kept at their base values. For each perturbation, the optimization problem is solved again and the percentage changes in the optimal positive-inventory time, cycle length, maximum inventory, maximum backlog, order quantity and average total cost are recorded.

Sensitivity analysis for demand and price parameters at -50% , -25% , $+25\%$ and $+50\%$.

Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
a	-50	+20.893	+21.817	-36.523	-22.788	-30.263	-22.790
a	-25	+8.579	+8.918	-17.017	-10.467	-14.032	-10.468
a	+25	-6.508	-6.736	+15.336	+9.285	+12.578	+9.283
a	+50	-11.744	-12.118	+29.365	+17.720	+24.057	+17.722
η	-50	+17.808	+18.573	-29.969	-19.391	-25.148	-19.393
η	-25	+8.773	+9.117	-16.188	-10.377	-13.539	-10.378
η	+25	-8.445	-8.732	+18.976	+11.952	+15.774	+11.953
η	+50	-16.559	-17.057	+41.079	+25.723	+34.080	+25.726
C	-50	+18.115	+18.893	-44.310	-23.267	-34.719	-23.271
C	-25	+6.599	+6.849	-16.232	-8.935	-12.906	-8.936
C	+25	-4.550	-4.714	+11.977	+6.729	+9.585	+6.732
C	+50	-8.022	-8.292	+21.619	+12.221	+17.336	+12.222
α	-50	+0.890	+0.323	+0.951	-0.187	+0.432	-0.187
α	-25	+0.443	+0.161	+0.473	-0.093	+0.215	-0.093
α	+25	-0.415	-0.149	-0.441	+0.091	-0.198	+0.091
α	+50	-0.838	-0.302	-0.892	+0.181	-0.403	+0.181

Sensitivity analysis for Weibull deterioration, backlogging and holding-cost parameters.

Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
β	-50	-1.133	-0.125	-0.706	+1.194	+0.160	+0.946

Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
β	-25	-1.133	-0.387	-1.151	+0.316	-0.482	+0.316
β	+25	+0.640	+0.222	+0.658	-0.167	+0.282	-0.168
β	+50	+1.055	+0.371	+1.098	-0.259	+0.479	-0.261
γ	-50	-0.625	-0.193	-0.583	+0.242	-0.207	+0.242
γ	-25	-0.303	-0.095	-0.287	+0.112	-0.105	+0.112
γ	+25	+0.309	+0.103	+0.307	-0.094	+0.124	-0.096
γ	+50	+0.587	+0.198	+0.588	-0.174	+0.241	-0.175
ε	-50	-0.011	+0.008	-0.014	+0.050	+0.015	-0.010
ε	-25	-0.006	+0.004	-0.007	+0.025	+0.007	-0.005
ε	+25	+0.005	-0.004	+0.006	-0.025	-0.008	+0.005
ε	+50	+0.011	-0.008	+0.013	-0.050	-0.016	+0.010
h_0	-50	+30.143	+9.310	+40.135	-13.832	+15.537	-13.831
h_0	-25	+12.920	+3.818	+16.245	-5.989	+6.111	-5.986
h_0	+25	-10.131	-2.830	-11.918	+4.757	-4.318	+4.756
h_0	+50	-18.349	-5.032	-21.136	+8.645	-7.562	+8.642
h_1	-50	+1.165	+0.393	+1.414	-0.343	+0.613	-0.345
h_1	-25	+0.582	+0.197	+0.705	-0.169	+0.307	-0.171
h_1	+25	-0.553	-0.184	-0.668	+0.171	-0.286	+0.168
h_1	+50	-1.109	-0.371	-1.337	+0.334	-0.576	+0.334

Sensitivity analysis for cost parameters.

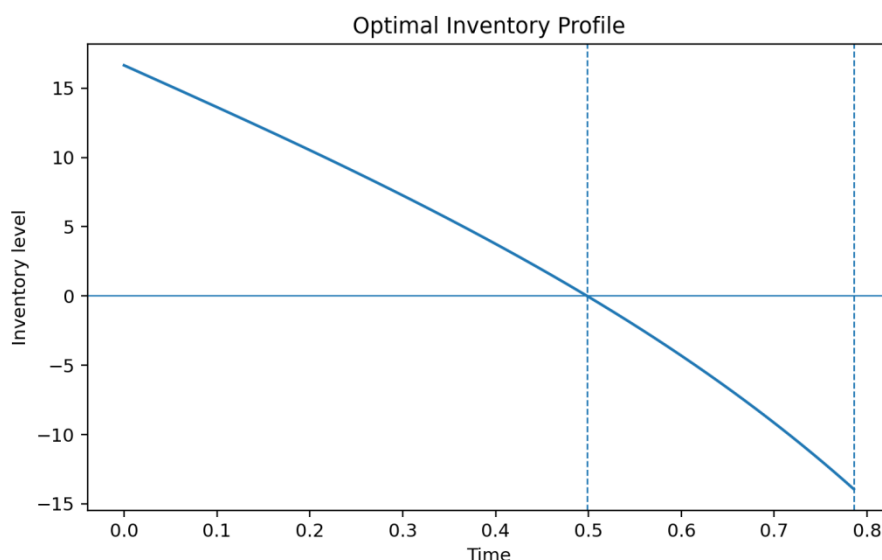
Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
Parameter	Change	t_1^* (%)	T^* (%)	W (%)	B (%)	Q (%)	TC^* (%)
c_d	-50	+0.700	+0.252	+0.848	-0.153	+0.392	-0.153
c_d	-25	+0.349	+0.126	+0.423	-0.076	+0.195	-0.076
c_d	+25	-0.325	-0.116	-0.393	+0.075	-0.180	+0.075
c_d	+50	-0.659	-0.236	-0.796	+0.148	-0.366	+0.149
c_s	-50	-15.807	+13.937	-18.324	+73.524	+23.539	-13.131
c_s	-25	-6.015	+5.142	-7.155	+26.408	+8.142	-5.157
c_s	+25	+4.103	-3.380	+5.023	-17.076	-5.050	+3.627
c_s	+50	+7.075	-5.781	+8.738	-29.087	-8.503	+6.323
c_l	-50	-0.014	+0.010	-0.017	+0.055	+0.016	-0.011
c_l	-25	-0.008	+0.005	-0.009	+0.027	+0.007	-0.006
c_l	+25	+0.007	-0.005	+0.008	-0.028	-0.008	+0.006
c_l	+50	+0.014	-0.011	+0.017	-0.055	-0.016	+0.011
K	-50	-19.743	-20.318	-22.663	-33.601	-27.649	-33.598
K	-25	-8.370	-8.655	-9.893	-15.869	-12.617	-15.867
K	+25	+6.648	+6.900	+8.200	+14.681	+11.154	+14.681
K	+50	+12.141	+12.632	+15.229	+28.521	+21.287	+28.521

The results in Tables 2–4 provide the local response of the optimized policy to both moderate and relatively large

perturbations. The numerical patterns are specific to the selected parameter set and are not interpreted as universal rankings of parameter importance.

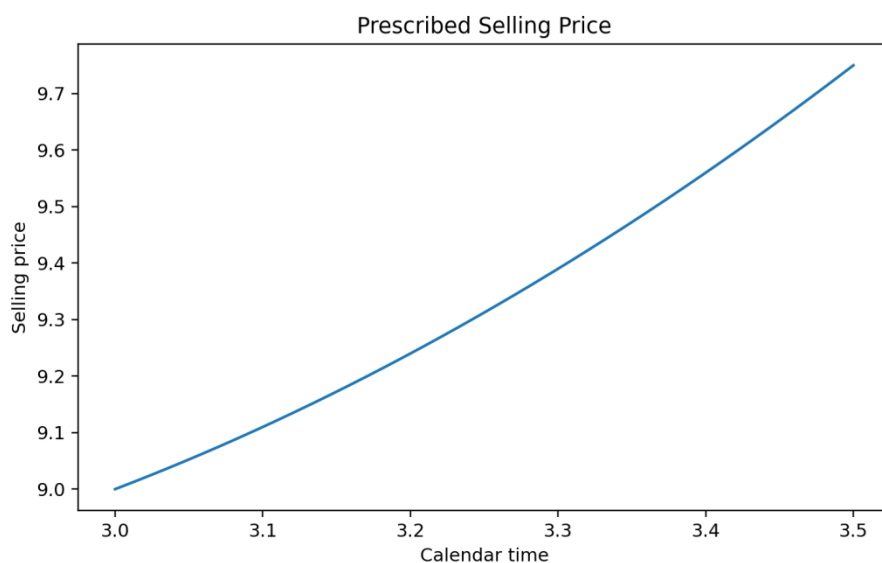
Graphical Representation and Interpretation

The graphical analysis begins with the optimal inventory profile, the prescribed price trajectory and the resulting demand profile. In addition, a separate sensitivity figure is provided for each model parameter. Each parameter-specific figure contains two panels: the upper panel shows the effects on t_1^* and T^* , while the lower panel shows the effects on W , B , Q and TC^* . This presentation makes the influence of each parameter visible independently.



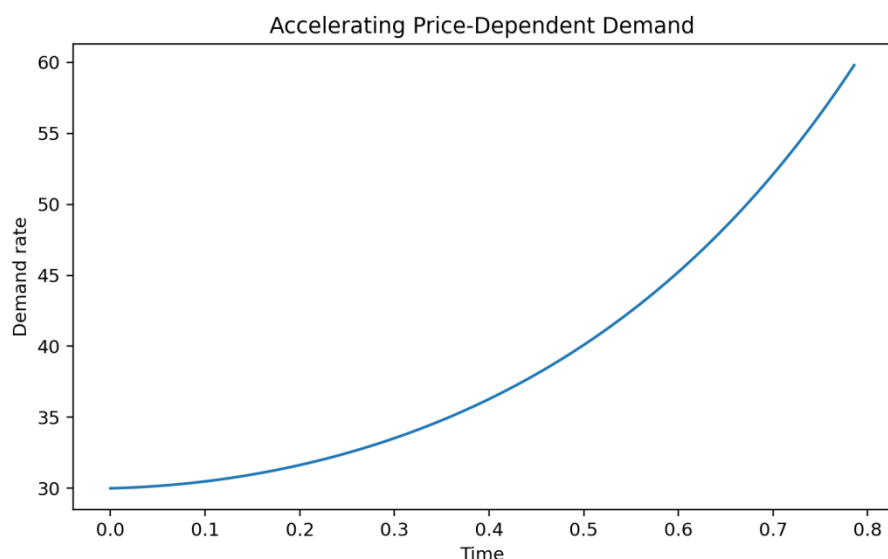
Optimal inventory profile.

Figure 1 illustrates the positive-inventory and shortage portions of the optimal cycle.



Prescribed quadratic selling-price trajectory.

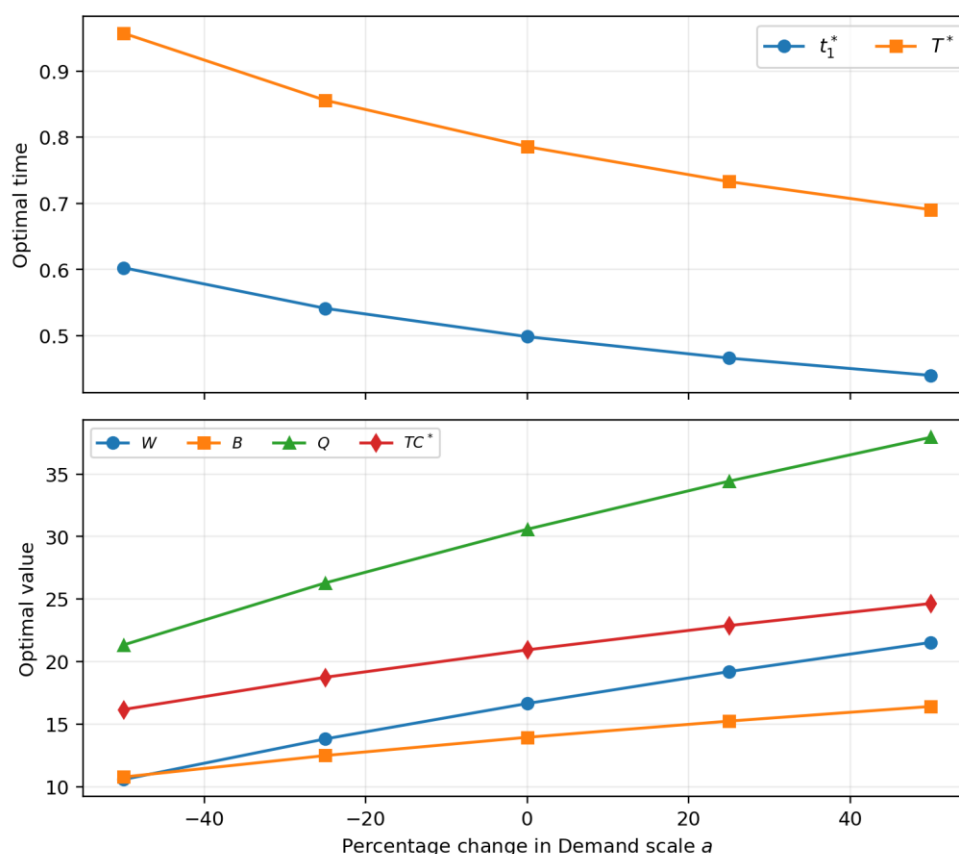
Figure 2 shows the prescribed selling-price path used in the demand function.



Accelerating exponential price-dependent demand.

Figure 3 shows the resulting demand profile produced by the accelerating time factor and positive price dependence.

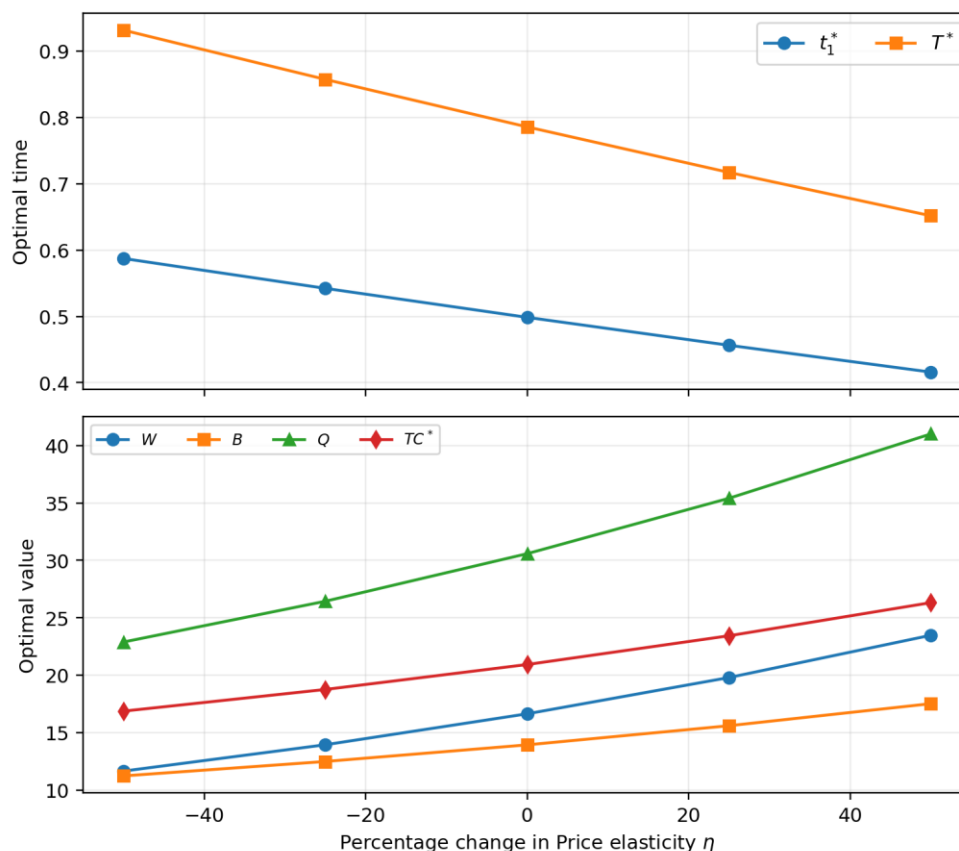
Effect of Demand scale a on optimal inventory decisions and cost



Separate sensitivity of optimal decisions and average total cost to demand scale a .

Figure 4 presents the separate response to demand scale a . Increasing a raises demand, shortening the optimized times and increasing the inventory, backlog, order quantity and average cost in this numerical study.

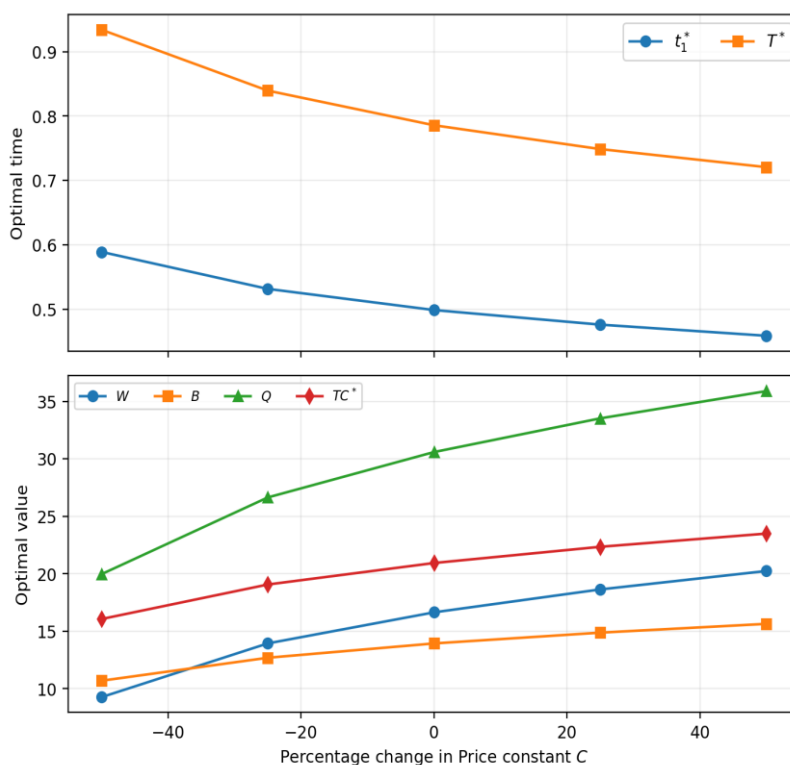
Effect of Price elasticity η on optimal inventory decisions and cost



Separate sensitivity of optimal decisions and average total cost to price elasticity η .

Figure 5 presents the separate response to price elasticity η . Increasing η strengthens the positive price response and produces shorter optimized times and larger inventory and order quantities, with a higher average cost.

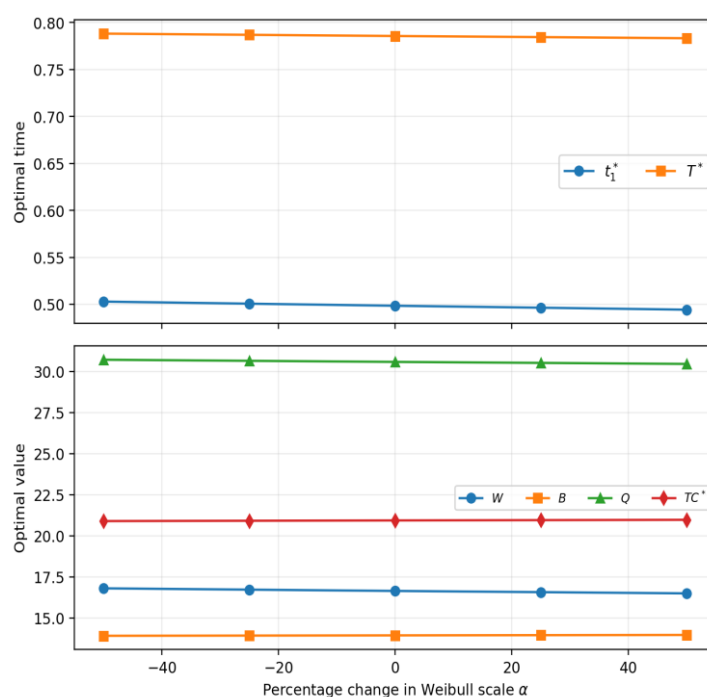
Effect of Price constant C on optimal inventory decisions and cost



Separate sensitivity of optimal decisions and average total cost to price constant C .

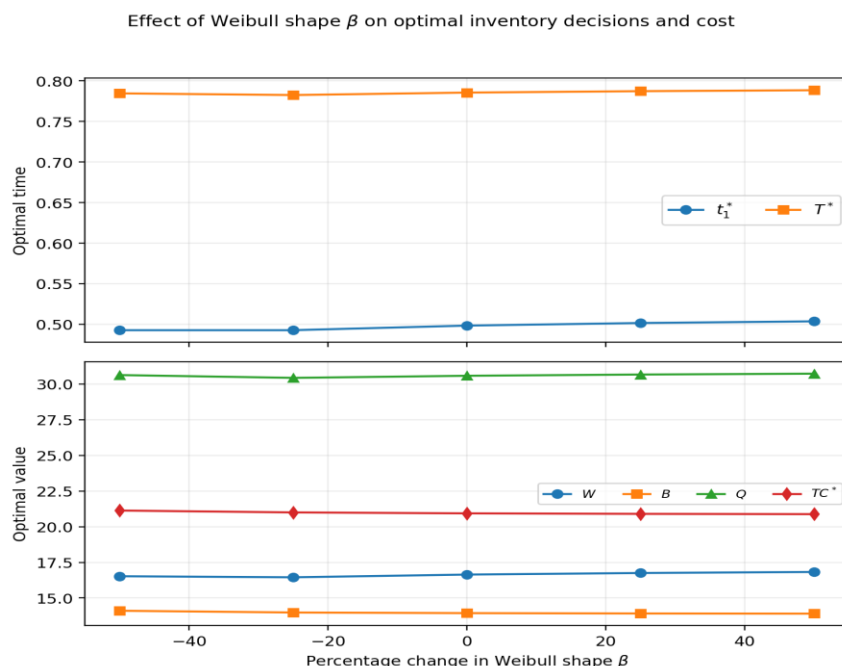
Figure 6 presents the separate response to price constant C . Changing C changes the price level and hence price-dependent demand; the optimized times and inventory quantities respond accordingly and the average cost rises over the tested range.

Effect of Weibull scale α on optimal inventory decisions and cost



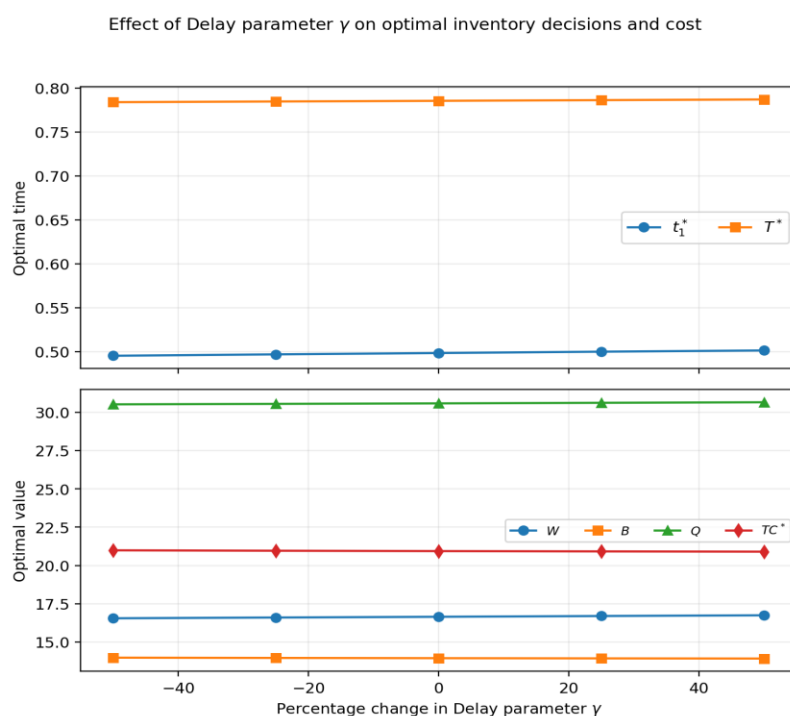
Separate sensitivity of optimal decisions and average total cost to Weibull scale parameter α .

Figure 7 presents the separate response to Weibull scale parameter α . Changing α changes deterioration intensity after the delay. The local effects on the optimized decisions are small in this example.



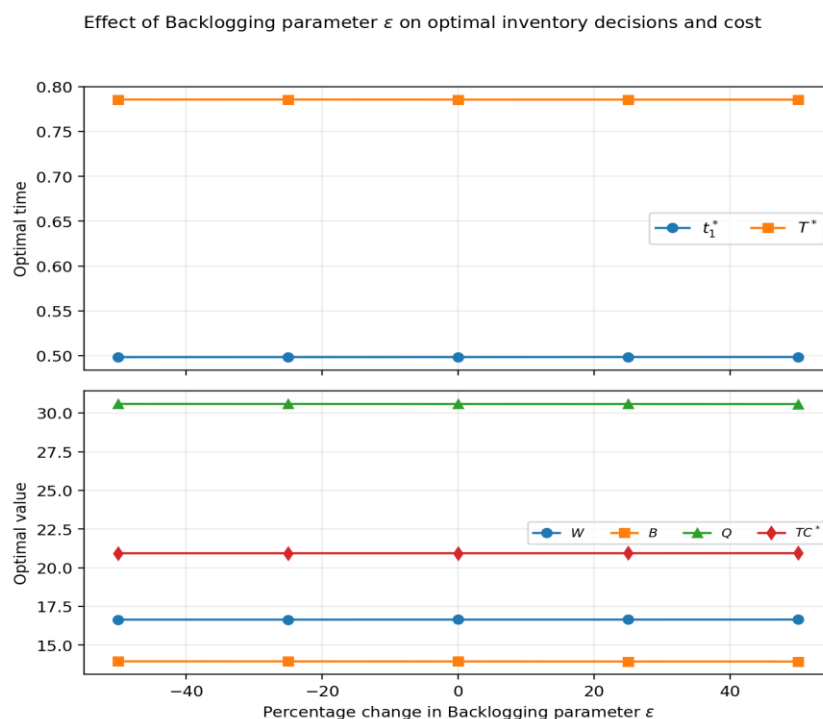
Separate sensitivity of optimal decisions and average total cost to Weibull shape parameter β .

Figure 8 presents the separate response to Weibull shape parameter β . Changing β changes the shape of the Weibull deterioration rate. The resulting local changes in the optimized quantities and cost are modest.



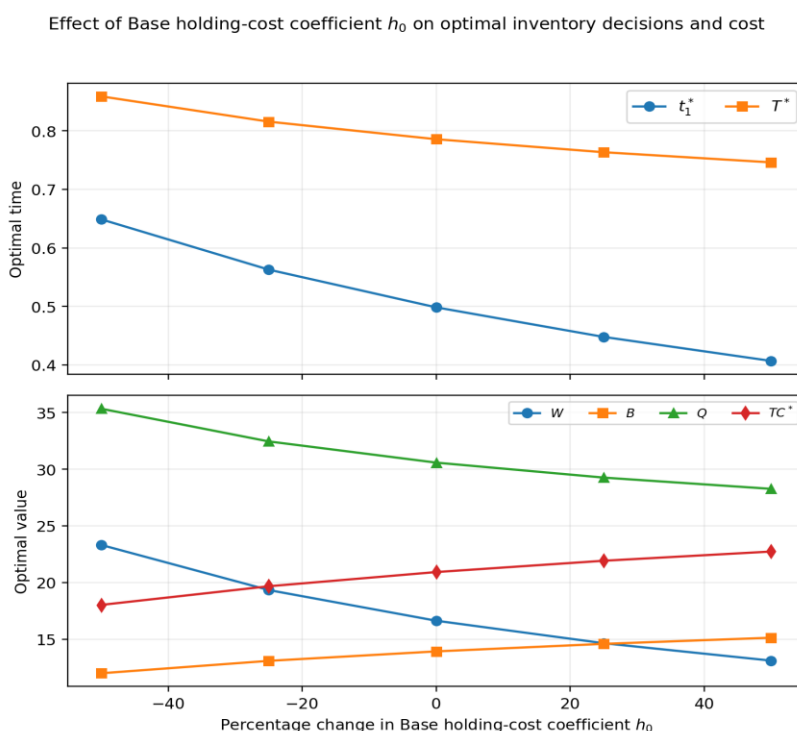
Separate sensitivity of optimal decisions and average total cost to delay parameter γ .

Figure 9 presents the separate response to delay parameter γ . Changing γ shifts the onset of deterioration. The optimized decisions and average cost show only moderate local variation.



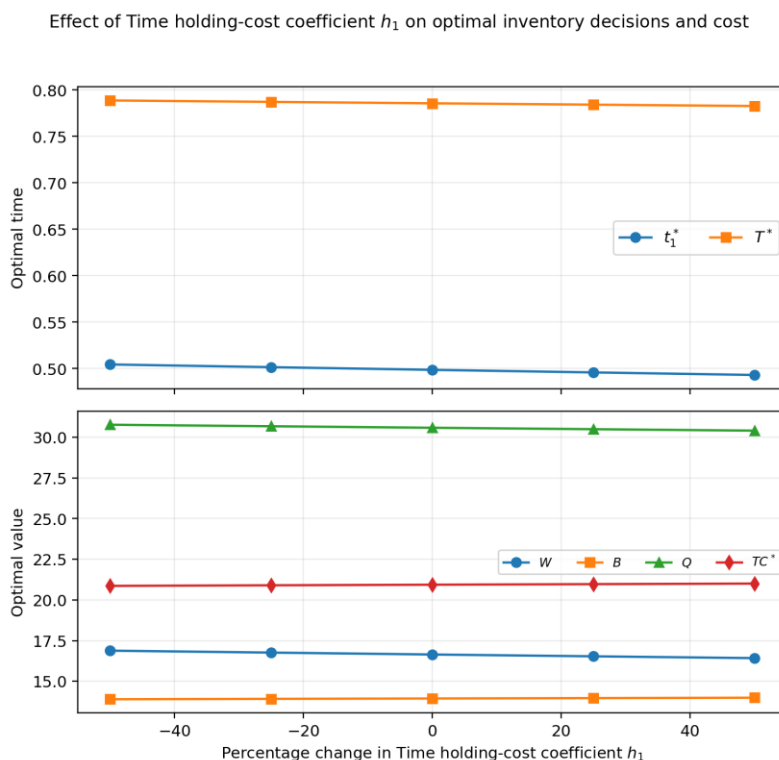
Separate sensitivity of optimal decisions and average total cost to backlogging parameter ε .

Figure 10 presents the separate response to backlogging parameter ε . Changing ε changes the backlogging fraction during shortage. Because the base value is small, its local effect on the optimized results is very small.



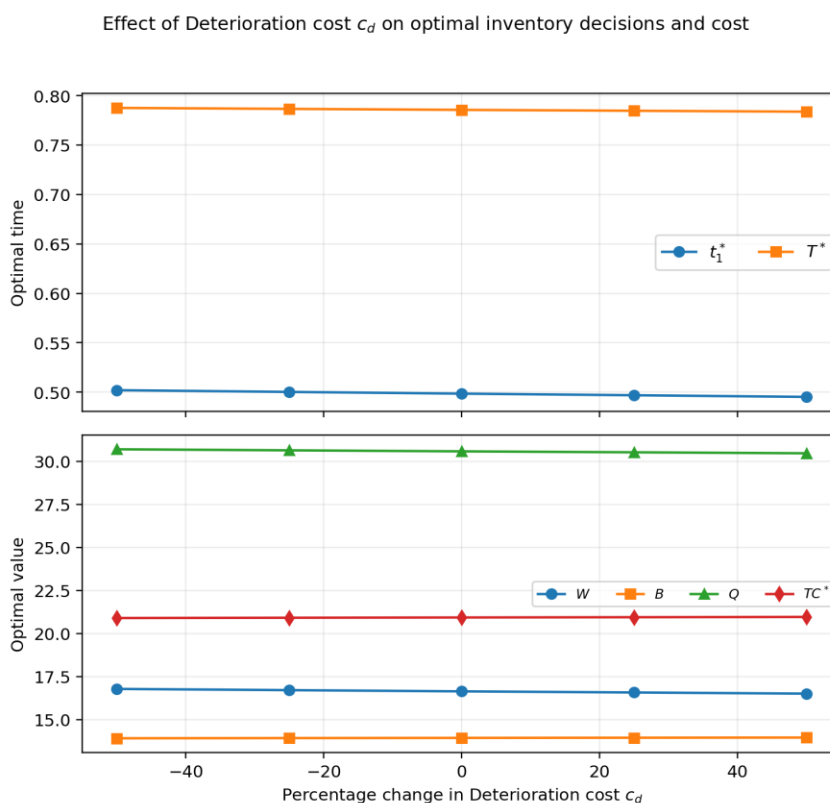
Separate sensitivity of optimal decisions and average total cost to base holding-cost coefficient h_0 .

Figure 11 presents the separate response to base holding-cost coefficient h_0 . Increasing h_0 raises the carrying cost of positive inventory. The optimized positive-inventory time and maximum inventory decrease, while average cost increases.



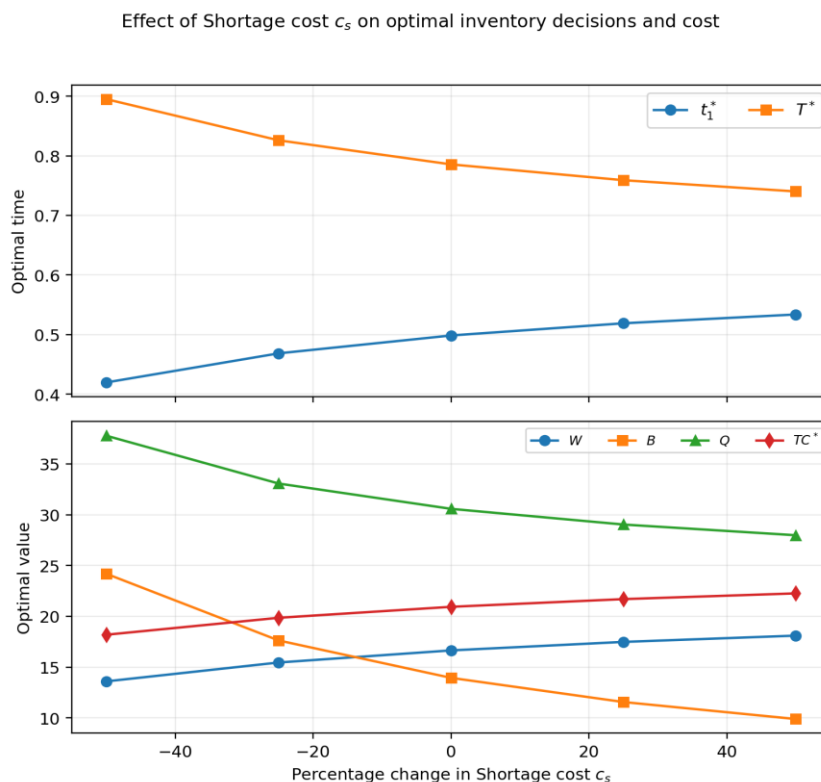
Separate sensitivity of optimal decisions and average total cost to time-dependent holding-cost coefficient h_1 .

Figure 12 presents the separate response to time-dependent holding-cost coefficient h_1 . Increasing h_1 raises the time-dependent holding charge. The optimized decisions show small local changes and average cost increases.



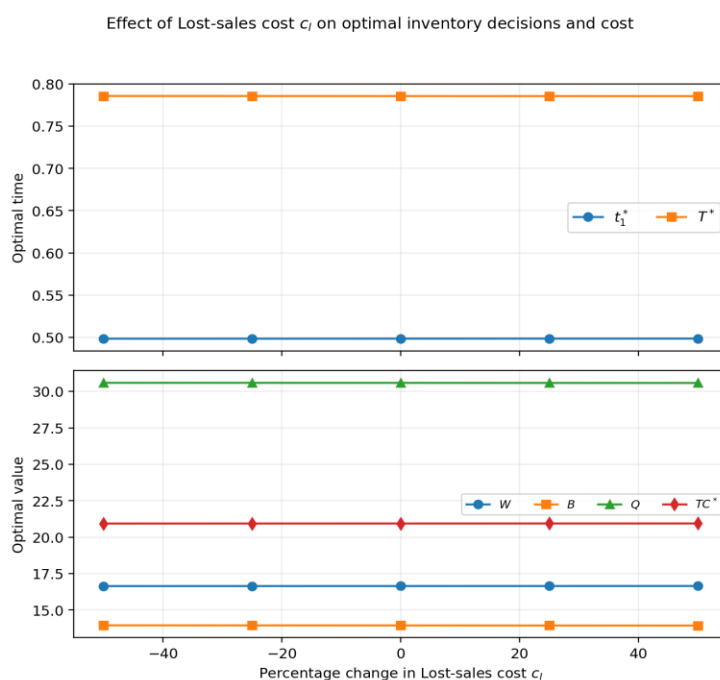
Separate sensitivity of optimal decisions and average total cost to deterioration cost coefficient c_d .

Figure 13 presents the separate response to deterioration cost coefficient c_d . Increasing c_d raises the cost assigned to deterioration. The optimized policy changes only slightly, while average cost increases.



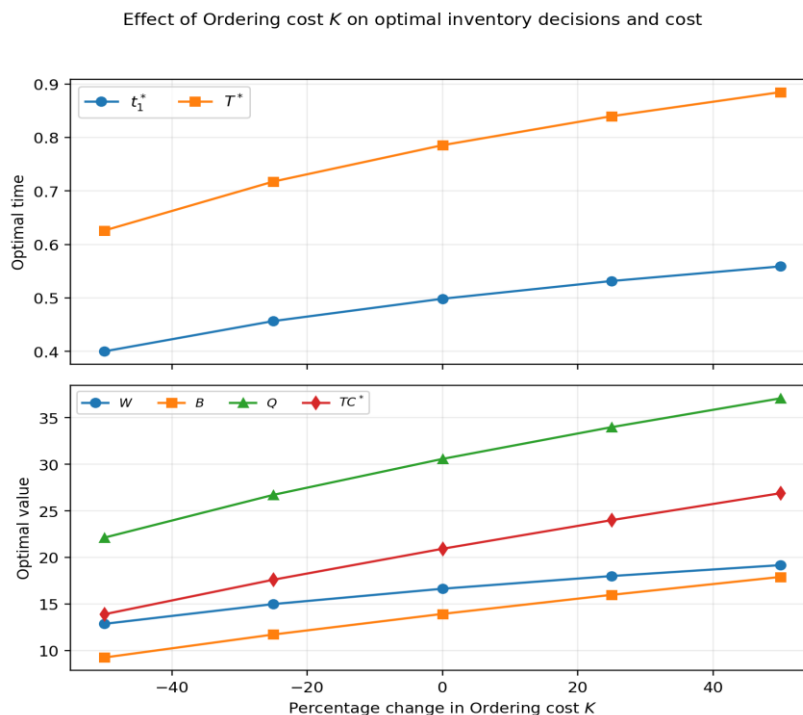
Separate sensitivity of optimal decisions and average total cost to shortage cost coefficient c_s .

Figure 14 presents the separate response to shortage cost coefficient c_s . Increasing c_s raises the cost of negative inventory. The shortage portion is reduced in the optimized cycle and average cost increases.



Separate sensitivity of optimal decisions and average total cost to lost-sales cost coefficient c_l .

Figure 15 presents the separate response to lost-sales cost coefficient c_l . Increasing c_l raises the cost of lost demand. Its numerical effect is small because the lost-sales component is small at the selected backlogging parameter.



Separate sensitivity of optimal decisions and average total cost to ordering cost K .

Figure 16 presents the separate response to ordering cost K . Increasing K raises the cost of each replenishment. The optimized cycle length and positive-inventory time increase, together with the order quantity and average cost.

Conclusion

A deterministic inventory model has been developed for deteriorating products with an accelerating exponential demand component e^{t^2} , price-dependent demand, a prescribed quadratic price trajectory, delayed Weibull deterioration, time-dependent holding cost and partial backlogging. The model is deliberately expressed with a clear distinction between calendar time and model time so that the six-month price calibration is mathematically explicit.

For the selected parameter set, the positive-inventory and shortage differential equations were solved using their boundary conditions, and the average total cost was constructed from ordering, holding, deterioration, shortage and lost-sales costs. Numerical optimization produced $t_1^* = 0.498554$ and $T^* = 0.785678$, with maximum inventory $W = 16.646351$, maximum backlog $B = 13.940716$, order quantity $Q = 30.587067$, and optimized average total cost approximately 20.936153.

The sensitivity analysis over $\pm 50\%$ changes demonstrates that demand, price and major cost parameters can alter the optimized policy appreciably, whereas some deterioration and backlogging parameters have smaller local effects in the chosen numerical example. The graphical analysis provides complementary insight into the stock trajectory, price movement, accelerating demand and parameter sensitivity.

Future work may make price an endogenous decision variable, replace the prescribed quadratic price path with a piecewise six-month pricing policy, introduce inflation, fuzzy parameters,

preservation technology or stochastic demand, or establish stronger global optimality results under additional analytical conditions.

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