

International Research Journal of Management Science & Technology



ISSN 2250 – 1959(Online)
2348 – 9367 (Print)

An Internationally Indexed Peer Reviewed & Refereed Journal

www.IRJMST.com
www.isarasolutions.com

Published by iSaRa Solutions

Some Generalized New Sequence Spaces

Naveen Kumar Srivastava

Department of Mathematics

St. Andrew’s College, Gorakhpur - 273001, Uttar Pradesh, India

email : srivastava.drnaveenkumar@gmail.com

Abstract

In the present paper we introduce some new sequence spaces defined by a Musielak-Orlicz function on semi-normed spaces. We make an effort to study some topological properties and inclusion relations between these spaces. The study of sequence spaces over n-normed spaces has also been initiated in this paper.

Keywords : Orlicz function, Musielak-Orlicz function, modulus function, paranorm space, sequence space, n-normed spaces.

1. Introduction and Preliminaries

Definition 1.1 : Let X be a linear metric space. A function $p : X \rightarrow \mathbb{R}$ is called paranorm, if

- (1) $p(x) \geq 0$ for all $x \in X$;
- (2) $p(-x) = p(x)$ for all $x \in X$;
- (3) $p(x + y) \leq p(x) + p(y)$ for all $x, y \in X$; and
- (4) if (μ_n) is a sequence of scalars with $\mu_n \rightarrow \mu$ as $n \rightarrow \infty$ and $(\lambda_n x_n)$ is a sequence of vectors with $p(\lambda x_n - \lambda x) \rightarrow 0$ as $n \rightarrow \infty$, then $p(\mu x_n - \mu x) \rightarrow 0$ as $n \rightarrow \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X, p) is called a total paranormed space.

It is well known that the metric of any linear metric space is given by some total paranorm, see Wilansky (1984), Theorem 10.4.2, 183.

Definition 1.2 : An Orlicz function M is a function, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$.

Lindenstrauss and Tzafriri (1971) used the idea of Orlicz function to define the following sequence space:

Let w be the space of all real or complex sequences $x = (x_k)$. Then,

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|\lambda_k x_k|}{\rho}\right) < \infty, \text{ for all } \rho > 0 \right\}, \quad \dots(1)$$

which is called as an Orlicz sequence space. The space ℓ_M is a Banach space with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|\lambda_k x_k|}{\rho}\right) \leq 1 \right\}. \quad \dots(2)$$

It is shown by Lindenstrauss and Tzafriri (1971) that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p ($p \geq 1$). An Orlicz function M satisfies Δ_2 -condition if and only if for any constant $L > 1$ there exists a constant $K(L)$ such that $M(Lu) \leq K(L)M(u)$, for all values of $u \geq 0$. An Orlicz function M can always be represented in the following integral form :

$$M(x) = \int_0^x \eta(t)dt, \quad \dots(3)$$

where η is known as the kernel of M , is right differentiable for $t \geq 0$, $\eta(0) = 0$, $\eta(t) > 0$, η is non-decreasing and $\eta(t) \rightarrow \infty$ as $t \rightarrow \infty$.

Definition 1.3: A sequence $\mathcal{M} = (M_k)$ of Orlicz functions is called a Musielak- Orlicz function, see Maligranda (1989), and Musielak (1983). A sequence $\mathcal{N} = (N_k)$ defined by

$$N_k(v) = \sup\{|v|u - M_k(u) : u \geq 0\}, \quad k = 1, 2, \dots \quad \dots(4)$$

is called the complementary function of a Musielak-Orlicz function \mathcal{M} . For a given Musielak-Orlicz function \mathcal{M} , the Musielak-Orlicz sequence space $t_{\mathcal{M}}$ and its subspace $h_{\mathcal{M}}$ are defined as follows :

$$t_{\mathcal{M}} = \{x \in w : I_{\mathcal{M}}(c \lambda x) < \infty \text{ for some } c > 0\}, \quad \dots(5)$$

$$h_{\mathcal{M}} = \{x \in w : I_{\mathcal{M}}(c \lambda x) < \infty \text{ for all } c > 0\}, \quad \dots(6)$$

where $I_{\mathcal{M}}$ is a convex modular defined by

$$I_{\mathcal{M}}(\lambda x) = \sum_{k=1}^{\infty} M_k(\lambda_k x_k), \quad \lambda x = (\lambda_k x_k) \in t_{\mathcal{M}}. \quad \dots(7)$$

We consider $t_{\mathcal{M}}$ equipped with the Luxemburg norm

$$\|x\| = \inf \left\{ k > 0 : I_{\mathcal{M}} \left(\frac{\lambda x}{k} \right) \leq 1 \right\} \quad \dots(8)$$

or equipped with the Orlicz norm

$$\|x\|^0 = \inf \left\{ \frac{1}{k} > (1 + I_{\mathcal{M}}(k \lambda x)) : k > 0 \right\} \quad \dots(9)$$

The notion of difference sequence spaces was introduced by Kizmaz (1981), who studied the difference sequence $\ell_{\infty}(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. The notion was further generalized by Et and Colak (1995) by introducing the spaces $\ell_{\infty}(\Delta^n)$, $c(\Delta^n)$ and $c_0(\Delta^n)$. Let m, n be non-negative integers, then for Z a given sequence space, we have

$$Z(\Delta_m^n) = \{x = (\lambda_k x_k) \in w : (\Delta_m^n \lambda_k x_k) \in Z\}, \quad \dots(10)$$

for $Z=c, c_0$ and ℓ_{∞} , where $\Delta_m^n \lambda x = (\Delta_m^n \lambda_k x_k) = (\Delta_m^{n-1} \lambda_k x_k - \Delta_m^{n-1} \lambda_{k+m} x_{k+m})$ and $\Delta_m^0 \lambda_k x_k = \lambda_k x_k$, for all $k \in \mathbb{N}$, which is equivalent to the following binomial representation:

$$\Delta_m^n \lambda_k x_k = \sum_{v=0}^n (-1)^v \binom{n}{v} \lambda_{k+mv} x_{k+mv} \quad \dots(11)$$

Taking $m = 1$, we get the spaces $\ell_{\infty}(\Delta^n)$, $c(\Delta^n)$ and $c_0(\Delta^n)$ studied by Et and Colak (1995). Taking $m = n = 1$, we get the spaces $\ell_{\infty}(\Delta)$, $c(\Delta)$ introduced and studied by Kizmaz (1981). The difference sequence spaces were also discussed in Altinok et.al., (2006), Isik (2004), and Tripathy et.al., (2006). For more details about sequence spaces are [Altinok (2008), Et et.al., (2006), Isik et.al., (2013), Murasaleen (1983), Musaleen et.al., (1999), Raj et.al., (2010), 2011), Raj and Sharma (2011, 2011) Savas (2004)] and references therein.

Let X be a complex linear space and (X, q) be a semi normed space with semi-norm q . By $S(X)$ we denote the linear space of all sequences $x = (\lambda_k x_k)$ with $(\lambda_k x_k) \in X$ and the usual coordinate wise operations :

$$\alpha x = (\alpha \lambda_k x_k)$$

and
$$x + y = (\lambda_k x_k + \lambda_k y_k) \dots(12)$$

for each $\alpha \in C$, where C denotes the set of all complex numbers. A study of sequence spaces $\ell_M(p, q, s)$ on semi normed spaces one can see in Bektas and Altin (2003).

The following inequality will be used throughout the paper. If $0 \leq p_k \leq \sup p_k = H, K = \max(1, 2^{H-1})$, then

$$|a_k + b_k|^{p_k} \leq K \{ |a_k|^{p_k} + |b_k|^{p_k} \}, \text{ for all } k \text{ and } a_k, b_k \in C \dots(13)$$

Also $|a_k|^{p_k} \max(1, |\alpha|^H)$, for all $\alpha \in C$.

2. Sequence Spaces on Semi-Normed Spacesefined by a Musielak-Orlicz Function

Let $\mu = (\mu_k)$ be a scalar sequence and $x \in S(X)$ then we shall write $\mu x = (\mu_k x_k)$. Let U be the set of all sequences $u = (u_k)$ such that $u_k \neq 0, p = (p_k)$ be a sequence of strictly positive real numbers and $\mathcal{M} = (M_k)$ be a Musielak-Orlicz function. In the present section we define the sequence spaces:

$$\ell_{\mathcal{M}}(\Delta_m^n, \lambda, u, p, q, s) =$$

$$\left\{ x \in S(X) : \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho} \right) \right) \right]^{p_k} < \infty, \text{ for some } \rho > 0 \text{ and } s \geq 0 \right\} \dots(14)$$

If we take $p = (p_k) = 1$, for all $k \in N$, then

$$\ell_{\mathcal{M}}(\Delta_m^n, \lambda_k, u, q, s) =$$

$$\left\{ x \in S(X) : \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0 \text{ and } s \geq 0 \right\} \dots(15)$$

If we take $s = 0$, we have

$$\ell_{\mathcal{M}}(\Delta_m^n, \lambda_k, u, p, q) =$$

$$\left\{ x \in S(X) : \sum_{k=1}^{\infty} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho} \right) \right) \right]^{p_k} < \infty, \text{ for some } \rho > 0 \right\} \dots(16)$$

If we take $p = (p_k) = 1$, for all k and $s = 0$, we get

$$\ell_{\mathcal{M}}(\Delta_m^n, \lambda_k, u, q) =$$

$$\left\{ x \in S(X) : \sum_{k=1}^{\infty} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho} \right) \right) \right] < \infty, \text{ for some } \rho > 0 \right\} \dots(17)$$

If we take $s = 0, q(x) = |x|$ and $X = C$, we have

$$\ell \mathcal{M}(\Delta_m^n, \lambda_k, u, p) = \left\{ x \in S(X) : \sum_{k=1}^{\infty} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho} \right) \right) \right]^{p_k} < \infty, \text{ for some } \rho > 0 \right\} \dots(18)$$

The main purpose of this section is to study some topological properties and some inclusion relations between the sequence spaces which we have defined above.

Theorem 2.1 : Let $\mathcal{M} = (M_k)$ be a Musielak-Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers. Then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is a linear space over the field of complex number C .

Proof : Let $x = (\lambda_k x_k), y = (\lambda_k y_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ and $\alpha, \beta \in C$. Then, there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n x_k}{\rho_1} \right) \right) \right]^{p_k} < \infty \dots(19)$$

and $\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\lambda_k \mu_k \Delta_m^n y_k}{\rho_2} \right) \right) \right]^{p_k} < \infty \dots(20)$

Let $\rho_3 = \max(2|\alpha| \rho_1, 2|\beta| \rho_2)$. Since $\mathcal{M} = (M_k)$ is non-decreasing, convex and q is a semi-norm by using inequality (13), we have

$$\begin{aligned} & \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k (\alpha \Delta_m^n \lambda_k x_k + \beta \Delta_m^n \lambda_k y_k)}{\rho_3} \right) \right) \right]^{p_k} \\ & \leq \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\alpha u_k \Delta_m^n \lambda_k x_k}{\rho_3} \right) + q \left(\frac{\beta u_k \Delta_m^n \lambda_k y_k}{\rho_3} \right) \right) \right]^{p_k} \\ & \leq \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1} \right) \right) + M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2} \right) \right) \right]^{p_k} \\ & \leq \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1} \right) \right) + M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2} \right) \right) \right]^{p_k} \\ & \leq K \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \lambda_k \Delta_m^n x_k}{\rho_1} \right) \right) \right]^{p_k} + K \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \lambda_k \Delta_m^n y_k}{\rho_2} \right) \right) \right]^{p_k} \dots(21) \end{aligned}$$

Thus, $\alpha x + \beta y \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Hence, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is a linear space.

Theorem 2.2 : Suppose $\mathcal{M} = (M_k)$ is a Musielak-Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers. Then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is a paranormed space with the paranorm defined by

$$g(\lambda x) = \inf \left\{ \rho^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, n = 1, 2, 3, \dots \right\} \dots(22)$$

where $M = \max(1, \sup, p_k)$.

Proof.

- (i) Clearly, $g(\lambda x) \geq 0$, for $\lambda x = (\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Since, $M_k(0) = 0$, we get $g(0) = 0$.
- (ii) $g(-\lambda x) = g(\lambda x)$.
- (iii) Let $\lambda x = (\lambda_k x_k), \lambda y = (\lambda_k y_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Then there exist $\rho_1, \rho_2 > 0$ such that

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1} \right) \right) \right]^{p_k} \leq 1 \dots(23)$$

and $\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_2} \right) \right) \right]^{p_k} \dots(24)$

If $\rho = \rho_1 + \rho_2$, then by Minkowskis inequality we have

$$\begin{aligned} \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n (\lambda_k x_k + \lambda_k y_k)}{\rho} \right) \right) \right]^{p_k} &= \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k + u_k \Delta_m^n \lambda_k y_k}{\rho_1 + \rho_2} \right) \right) \right]^{p_k} \\ &\leq \left(\frac{\rho_1}{\rho_1 + \rho_2} \right) \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1} \right) \right) \right]^{p_k} + \\ &\quad \left(\frac{\rho_2}{\rho_1 + \rho_2} \right) \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_2} \right) \right) \right]^{p_k} \leq 1 \dots(25) \end{aligned}$$

and thus,

$$\begin{aligned} g(x + y) &= \inf \left\{ \rho^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k + u_k \Delta_m^n \lambda_k y_k}{\rho} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, \rho > 0 \right\} \\ &\leq \inf \left\{ (\rho_1)^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, \rho_1 > 0 \right\} \end{aligned}$$

$$\leq \inf \left\{ (\rho_2)^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho_2} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, \rho_2 > 0 \right\}$$

$$\leq g(x) + g(y) \quad \dots(26)$$

(iv) Finally, we prove that scalar multiplication is continuous. Let λ be any complex number,

$$g(\lambda x) = \inf \left\{ \rho^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\mu u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, n = 1, 2, 3, \dots \right\}$$

$$= \inf \left\{ |\mu| r^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} \right)^{\frac{1}{M}} \leq 1, s \geq 0, n = 1, 2, 3, \dots \right\} \quad \dots(27)$$

where $r = \frac{\rho}{|\mu|}$. Hence, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is a paranormed space.

Theorem 2.3 :

- (i) Let $0 < p_k < t_k < \infty$, for each $k \in \mathbb{N}$. Then, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, t, q)$,
- (ii) Let $0 < p_k < t_k < \infty$, for each $k \in \mathbb{N}$. Then, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, t)$,
- (iii) $\ell \mathcal{M}(\Delta_m^n, \lambda, u, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, q, s)$,
- (iv) $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$.

Proof :

(i) Let $\lambda x = (\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Then, there exists some $\rho > 0$ such that

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} < \infty, \quad \dots(28)$$

which implies that $M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \leq 1$ for sufficiently large value of k , say $k \geq m_0$ for some

fixed $m_0 \in \mathbb{N}$. Since $\mathcal{M} = (M_k)$ is non-decreasing, we get

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{t_k} < \infty, \quad \dots(29)$$

Thus, we have

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{t_k} \leq \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} < \infty \quad \dots(30)$$

This shows that $\lambda x = (\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Hence, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, t, q)$. Similarly, we can prove (ii), (iii) and (iv) in view of (i).

Corollary 2.4 :

- (i) If $0 < p_k \leq 1$, for all $k \in \mathbb{N}$, then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, q)$.
- (ii) If $p_k \leq 1$, for all $K \in \mathbb{N}$, then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q)$.

Proof :

- (i) If we take $t_k = 1$ for all $k \in \mathbb{N}$, in Theorem 2.3(i), we get

$$\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, q) \quad \dots(31)$$

- (ii) If we take $p_k = 1$, for all $k \in \mathbb{N}$, in Theorem 2.3(i), we get

$$\ell \mathcal{M}(\Delta_m^n, \lambda, u, q) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q) \quad \dots(32)$$

Theorem 2.5 : The sequence space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is solid.

Proof : Let $\lambda x = (\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ i.e.,

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} < \infty \quad \dots(33)$$

Let (α_k) be a sequence of scalars such that $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$. Thus, we have

$$\sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{\alpha_k u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} \leq \sum_{k=1}^{\infty} k^{-s} \left[M_k \left(q \left(\frac{u_k \Delta_m^n \lambda_k x_k}{\rho} \right) \right) \right]^{p_k} < \infty \quad \dots(34)$$

This shows that $(\alpha_k \lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$, for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$ whenever $(\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$. Hence, the space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is a solid sequence space.

Theorem 2.6 : The sequence space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$ is monotone.

Proof : The proof is trivial so we omit it.

Corollary 2.7 :

- (i) Let $|u_k| \leq 1$ for all $k \in \mathbb{N}$. Then $\ell \mathcal{M}(\Delta_m^n, \lambda, p, q, s) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s)$.
- (ii) Let $|u_k| \geq 1$ for all $k \in \mathbb{N}$. Then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, q, s) \subseteq \ell \mathcal{M}(\Delta_m^n, \lambda, p, q, s)$.

Proof : It is obvious.

3. Sequence Spaces defined by a Musielak-Orlicz Function Over n-Normed Spaces

In this section we define some sequence spaces defined by a Musielak-Orlicz function over n-normed spaces. We also study some topological properties on these spaces. The concept of 2-normed

spaces was initially developed by Gähler (1965) in the mid of 1960's, while that of n-normed spaces can be found in Misiak (1989). Since, many others have studied this concept and obtained various results, see Gunawan (2001, 2001) and Gunawan and (Mashadi) (2001).

Let $n \in \mathbb{N}$ and X be a real vector space of dimension d , where $n \geq d$. A real valued function $\|\cdot, \dots, \cdot\|$ on X^n satisfying the following four conditions:

- (1) $\|x_1, x_2, \dots, x_n\| = 0$ if and only if x_1, x_2, \dots, x_n are linearly dependent,
- (2) $\|x_1, x_2, \dots, x_n\|$ is invariant under permutation,
- (3) $\|\alpha x_1, x_2, \dots, x_n\| = |\alpha| \|x_1, x_2, \dots, x_n\|$ for any $\alpha \in \mathbb{R}$ and
- (4) $\|x + x', x_2, \dots, x_n\| \leq \|x_1, x_2, \dots, x_n\| + \|x', x_2, \dots, x_n\|$ is called an n-norm on X , and the pair $(X, \|\cdot, \dots, \cdot\|)$ is called an n-normed space.

For example, we may take $x = \mathbb{R}^n$ being equipped with the n-norm $\|x_1, x_2, \dots, x_n\|_E =$ the volume of the n-dimensional parallelepiped spanned by the vectors x_1, x_2, \dots, x_n which may be given explicitly by the formula

$$\|x_1, x_2, \dots, x_n\|_E = \left| \det(x_{i,j}) \right| \quad \dots(35)$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \dots, n$ and E denote the Euclidean norm. Let $(X, \|\cdot, \dots, \cdot\|)$ be an n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, \dots, a_n\}$ be linearly independent set in X . Then, the following function $\|\cdot, \dots, \cdot\|_\infty$ on X^{n-1} defined by

$$\|x_1, x_2, \dots, x_{n-1}\|_\infty = \max\{\|x_1, x_2, \dots, x_{n-1}, a_i\| : i = 1, 2, \dots, n\} \quad \dots(36)$$

defines an $(n - 1)$ norm on X with respect to $\{a_1, a_2, \dots, a_n\}$.

A sequence (x_k) in an n-normed space $(X, \|\cdot, \dots, \cdot\|)$ is said to converge to some $L \in X$ in the n-norm if $\lim_{k \rightarrow \infty} \|\lambda_k x_k - L, z_1, \dots, z_{n-1}\| = 0$, for every $z_1, \dots, z_{n-1} \in X$.

A sequence (x_k) in an n-normed space $(X, \|\cdot, \dots, \cdot\|)$ is said to be Cauchy with respect to the n-norm if $\lim_{k,p \rightarrow \infty} \|\lambda_k x_k - \lambda_p x_p, z_1, \dots, z_{n-1}\| = 0$, for every $z_1, \dots, z_{n-1} \in X$.

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the n-norm. Any complete n-normed space is said to be n-Banach space.

In this section we define the sequence space:

$$\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|) = \left\{ x \in S(X) : \sum_{k=1}^{\infty} M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right)^{p_k} < \infty, \text{ for some } p > 0 \right\} \quad \dots(37)$$

Theorem 3.1 : Let $\mathcal{M} = (M_k)$ be a Musielak Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers. Then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is a linear space over the field of complex numbers \mathbb{C} .

Proof : Let $\lambda x = (\lambda_k x_k), y = (\lambda_k y_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ and $\alpha, \beta \in \mathbb{C}$. Then there exist positive numbers ρ_1 and ρ_2 such that

$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} < \infty \quad \dots(38)$$

and
$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \quad \dots(39)$$

Let $\rho_3 = \max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Since $\mathcal{M} = (M_k)$ is nondecreasing convex function, by using inequality (13) we have

$$\begin{aligned} & \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n (\alpha \lambda_k x_k + \beta \lambda_k y_k)}{\rho_3}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ &= \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{\alpha u_k \Delta_m^n \lambda_k x_k}{\rho_3}, z_1, \dots, z_{n-1} + \frac{\beta u_k \Delta_m^n \lambda_k y_k}{\rho_3}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ &\leq K \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ & \quad + K \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ &\leq K \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ & \quad + K \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} < \infty \quad \dots(40) \end{aligned}$$

Thus, $\alpha x + \beta y \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$. Therefore, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is a linear space.

Theorem 3.2 : If $\mathcal{M} = (M_k)$ is a Musielak-Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers, then $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is a paranorm defined by

$$g(\lambda x) = \inf \left\{ \rho \frac{p_n}{M} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \leq 1, n=1, 2, 3, \dots \right\} \quad \dots(41)$$

where $M = \max(1, \sup_k p_k)$.

Proof :

- (i) Clearly, $g(\lambda x) \geq 0$ for $\lambda x = (\lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$. Since $M_k(0) = 0$, we get $g(0) = 0$.
- (ii) $g(-\lambda x) = g(\lambda x)$.
- (iii) Let $\lambda x = (\lambda_k x_k), \lambda y = (\lambda_k y_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$. Then there exist $\rho_1, \rho_2 > 0$ such that

$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq 1 \quad \dots(42)$$

and

$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq 1 \quad \dots(43)$$

If $\rho = \rho_1 + \rho_2$, then by Minkowski's inequality, we have

$$\begin{aligned} \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} &= \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1 + \rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ &\leq \frac{\rho_1}{\rho_1 + \rho_2} \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \\ &\quad + \frac{\rho_2}{\rho_1 + \rho_2} \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq 1 \quad \dots(44) \end{aligned}$$

and thus,

$$\begin{aligned} g(\lambda x + \lambda y) &= \inf \left\{ (\rho_1 + \rho_2)^{\frac{p_n}{m}} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k + u_k \Delta_m^n \lambda_k y_k}{\rho_1 + \rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \right\} \\ &\leq \inf \left\{ (\rho_1)^{\frac{p_n}{m}} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho_1}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \right\} \\ &\quad + \inf \left\{ (\rho_2)^{\frac{p_n}{m}} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k y_k}{\rho_2}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \right\} \leq g(x) + g(y) \quad \dots(45) \end{aligned}$$

Finally, we prove that scalar multiplication is continuous. Let λ be any complex number

$$\begin{aligned}
 g(\lambda x) &= \inf \left\{ \rho^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \right\} \\
 &= \inf \left\{ (\lambda/r)^{\frac{p_n}{M}} : \left(\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \right)^{\frac{1}{m}} \right\} \dots(46)
 \end{aligned}$$

where $r = \frac{\rho}{|\mu|}$. Hence, $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is a paranormed space.

Theorem 3.3 : The sequence space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is solid.

Proof : Let $x = (x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$. Then,

$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} < \infty \dots(47)$$

Let (α_k) be a sequence of scalars such that $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$. Thus, we have

$$\sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{\alpha_k u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \leq \sum_{k=1}^{\infty} \left[M_k \left(\left\| \frac{u_k \Delta_m^n \lambda_k x_k}{\rho}, z_1, \dots, z_{n-1} \right\| \right) \right]^{p_k} \dots(48)$$

This shows that $(\alpha_k \lambda_k x_k) \in \ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$. Hence, the space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is a solid sequence space.

Theorem 3.4 : The sequence space $\ell \mathcal{M}(\Delta_m^n, \lambda, u, p, \|\cdot, \dots, \cdot\|)$ is monotone.

Proof : The proof is trivial so we omit it.

4. Applications of Sequence spaces

(a) Applications of Sequence Spaces in Matrix Theory : The theory of sequence spaces and their matrix maps has made remarkable advances in enveloping duality theory via unified techniques effecting matrix transformation from one sequence space into another. Thus, we have several important applications of the theory of sequence spaces. Apart from this, the theory of sequence spaces is powerful tool for obtaining positive results concerning Schauder basis and their associated type. Mathematicians, like Cesàro, Borel, Nörlund, Reisz and others have studied the general theory of matrix transformations motivated by special and classical results in summability theory. On the other hand, in most cases the general linear operators on one sequence space into another is actually given by an infinite matrix. The well-known German Mathematician O. Toeplitz, first observed in 1911, that the technique of linear space theory can be used to characterize matrix transformations, Later, the Banach-Steinhaus theorem and related results became useful tools in dealing with such problems.

(b) Applications of Sequence Spaces in Biomathematics : The relationship between sequences and binding properties of an aptamer for immunoglobulin (IgE) can be investigated using custom DNA microarrays. Single, double and some triple mutations of the aptamer sequence can be created to evaluate the importance of specific base composition on aptamer binding. The functional sequence space can be represented as a rugged landscape with sharp peaks defined by highly constrained base composition for more details see Katillius et al., (2007). One more interesting application is an Analysis of Peptides from known Proteins Clusterization in Sequence Space. A combinatorial sequence space (CSS) can be introduced to represent sequence as a set of overlapping K -tuples of some fixed length which correspond to a point in CSS. In Strelets et al. (1994) analyse clusterization of protein sequences in the CSS and to test various hypothesis about possible evolutionary basis of this clusterization. Possible applications of sequence spaces were also discussed in Strelets et al., (1994).

5. Conclusion

In this paper we have constructed some new sequence spaces defined by a Musielak-Orlicz function over semi normed spaces. We have also made an attempt to introduce some sequence spaces over n -normed spaces. We have studied some topological properties and interesting inclusion relations between these sequence spaces. The solutions obtained are potentially significant and important for the explanation of some practical physical problems. The method may also be applied to other sequence spaces.

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